

$$= \cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{5\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right)$$

$$A = \cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\pi - \frac{3\pi}{8}\right) + \cos^2\left(\pi - \frac{\pi}{8}\right)$$

$$A = \cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \left(-\cos\left(\frac{3\pi}{8}\right)\right)^2 + \left(-\cos\left(\frac{\pi}{8}\right)\right)^2$$

$$A = \cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{\pi}{8}\right)$$

$$A = 2\left(\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8}\right) = 2\left(\cos^2\frac{\pi}{8} + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right)$$

$$A = 2\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right) = 2$$

$$B = \sin^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right) + \sin^2\left(\frac{7\pi}{12}\right) + \sin^2\left(\frac{13\pi}{12}\right)$$

$$B = \sin^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{12}\right) + \sin^2\left(\frac{7\pi}{12}\right) + \sin^2\left(\frac{\pi}{2} + \frac{7\pi}{12}\right)$$

$$B = \sin^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{7\pi}{12}\right) + \cos^2\left(\frac{7\pi}{12}\right)$$

$$B = 1 + 1 = 2$$

$$C = \cos^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{11\pi}{12}\right) + 2\cos^2\left(\frac{5\pi}{12}\right)$$

$$C = \cos^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{5\pi}{12}\right) + \cos^2\left(\frac{5\pi}{12}\right) + \cos^2\left(\frac{11\pi}{12}\right)$$

$$C = \cos^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{12}\right) + \cos^2\left(\frac{5\pi}{12}\right) + \cos^2\left(\frac{\pi}{2} + \frac{5\pi}{12}\right)$$

$$C = \cos^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right) + \cos^2\left(\frac{5\pi}{12}\right) + \left(-\sin\left(\frac{5\pi}{12}\right)\right)^2$$

$$C = 1 + \cos^2\left(\frac{5\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right) = 1 + 1 = 2$$

$$D = \cos\left(\frac{\pi}{12}\right) \cdot \sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right) \cdot \cos\left(\frac{5\pi}{12}\right)$$

$$D = \cos\left(\frac{\pi}{12}\right) \cdot \sin\left(\frac{\pi}{2} + \frac{\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right) \cdot \cos\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$$

$$D = \cos\left(\frac{\pi}{12}\right) \cdot \cos\left(\frac{\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right) \cdot \sin\left(\frac{\pi}{12}\right)$$

$$D = 1$$

$$\begin{aligned}
 E &= \sin^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{3\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right) + \sin^2\left(\frac{7\pi}{12}\right) + \sin^2\left(\frac{9\pi}{12}\right) + \sin^2\left(\frac{11\pi}{12}\right) \\
 E &= \sin^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{7\pi}{12}\right) + \sin^2\left(\frac{3\pi}{12}\right) + \sin^2\left(\frac{9\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right) + \sin^2\left(\frac{11\pi}{12}\right) \\
 E &= \sin^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{\pi}{2} + \frac{\pi}{12}\right) + \sin^2\left(\frac{3\pi}{12}\right) + \sin^2\left(\frac{\pi}{2} + \frac{3\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right) + \sin^2\left(\frac{\pi}{2} + \frac{5\pi}{12}\right) \\
 E &= \sin^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{3\pi}{12}\right) + \cos^2\left(\frac{3\pi}{12}\right) + \sin^2\left(\frac{5\pi}{12}\right) + \cos^2\left(\frac{5\pi}{12}\right) \\
 E &= 3
 \end{aligned}$$

من خلال هذه الأمثلة متلاحظ أن تبسيط مثل هذه التعابير يعتمد على أمرين هامين :

= أولا ملاحظة العلاقة بين الأعداد للوجود مثلا: إذا اعتبرنا العددين $\frac{\pi}{12}$ و $\frac{5\pi}{12}$ فبعد جمعهما نجد: $\frac{\pi}{2}$ أي أن: $\frac{5\pi}{12} = \frac{\pi}{2} - \frac{\pi}{12}$

بينما إذا اعتبرنا العددين: $\frac{\pi}{7}$ و $\frac{8\pi}{7}$ فبعد طرحهما نجد: π أي أن: $\frac{8\pi}{7} = \pi + \frac{\pi}{7}$

= الأمر الثاني هو استعمال هذه الملاحظة و تطبيق قواعد الحساب المثلثي لأجل التبسيط و الحساب إن أمكن.

تمرين 2 :

$$\cos\left(\frac{26\pi}{3}\right) = \cos\left(\frac{24\pi + 2\pi}{3}\right) = \cos\left(8\pi + \frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\tan\left(\frac{85\pi}{4}\right) = \tan\left(\frac{84\pi + \pi}{4}\right) = \tan\left(21\pi + \frac{\pi}{4}\right) = \tan\left(20\pi + \pi + \frac{\pi}{4}\right) = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\sin\left(\frac{71\pi}{3}\right) = \sin\left(\frac{72\pi - \pi}{3}\right) = \sin\left(24\pi - \frac{\pi}{3}\right) = \sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{2}}{2}$$

قد نتساءل عن طريقة كتابة الأعداد أعلاه

الأمر ببساطة يعتمد على إجراء قسمة عادية، مثلا بعد قسمة 26 على 3 سيكون الخارج 8 والباقي 2 لذلك كتبنا

$26\pi = 3 \times 8\pi + 2\pi = 24\pi + 2\pi$ ، ويمكن أيضا كتابته على شكل فرق: $26\pi = 27\pi - \pi$ ، لكن يستحسن الحل الأول

لأنه يعطينا عددا زوجيا في الخارج بعد القسمة مما يسمح بتطبيق مباشر للخاصية: $\sin(x + 2k\pi) = \sin(x)$ أو

$\cos(x + 2k\pi) = \cos(x)$ أما بالنسبة لـ $\tan$ فالأمر ليس ضروريا لأن: $\tan(x + k\pi) = \tan(x)$

تمرين 3 :

$$\text{نعلم أن: } \sin^2\left(\frac{2\pi}{5}\right) + \cos^2\left(\frac{2\pi}{5}\right) = 1 \text{ منه: } \sin^2\left(\frac{2\pi}{5}\right) + \left(\frac{\sqrt{5}-1}{4}\right)^2 = 1 \text{ منه: } \sin^2\left(\frac{2\pi}{5}\right) + \frac{6-2\sqrt{5}}{16} = 1$$

$$\text{منه: } \sin^2\left(\frac{2\pi}{5}\right) = 1 - \frac{6-2\sqrt{5}}{16} = \frac{16-6+2\sqrt{5}}{16} = \frac{10+2\sqrt{5}}{16}$$

$$\text{وبما أن: } \frac{2\pi}{5} \in [0; \pi] \text{ فإن: } \sin\left(\frac{2\pi}{5}\right) \geq 0 \text{ ، بالتالي: } \sin\left(\frac{2\pi}{5}\right) = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\sin\left(\frac{3\pi}{5}\right) = \sin\left(\pi - \frac{2\pi}{5}\right) = \sin\left(\frac{2\pi}{5}\right) = \frac{\sqrt{10+2\sqrt{5}}}{4} \text{ , } \sin\left(\frac{-2\pi}{5}\right) = -\sin\left(\frac{2\pi}{5}\right) = -\frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\cos\left(\frac{\pi}{10}\right) = \cos\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) = \sin\left(\frac{2\pi}{5}\right) = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

تذكر دائما القاعدة الأساسية للحساب المثلثي: $\sin^2(x) + \cos^2(x) = 1$

تمرين 4 : $A(x) = \sin\left(x + \frac{\pi}{4}\right) + \cos\left(x + \frac{\pi}{4}\right)$	
1 لدينا : $A(-x) = \sin\left(-x + \frac{\pi}{4}\right) + \cos\left(-x + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2} - \left(-x + \frac{\pi}{4}\right)\right) + \sin\left(\frac{\pi}{2} - \left(-x + \frac{\pi}{4}\right)\right)$ $A(-x) = \cos\left(\frac{\pi}{2} + x - \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2} + x - \frac{\pi}{4}\right) = \cos\left(x + \frac{\pi}{4}\right) + \sin\left(x + \frac{\pi}{4}\right) = A(x)$	
2 لدينا : $A(\pi - x) = \sin\left(\pi - x + \frac{\pi}{4}\right) + \cos\left(\pi - x + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2} - \left(\pi - x + \frac{\pi}{4}\right)\right) + \cos\left(\frac{\pi}{2} - \left(\pi - x + \frac{\pi}{4}\right)\right)$ $A(\pi - x) = \cos\left(\frac{\pi}{2} - \pi + x - \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2} - \pi + x - \frac{\pi}{4}\right) = \cos\left(x + \frac{\pi}{4} - \pi\right) + \sin\left(x + \frac{\pi}{4} - \pi\right)$ $A(\pi - x) = \cos\left(x + \frac{\pi}{4} + \pi\right) + \sin\left(x + \frac{\pi}{4} + \pi\right) = -\cos\left(x + \frac{\pi}{4}\right) - \sin\left(x + \frac{\pi}{4}\right) = -A(x)$	
في كلا السؤالين استعملنا القواعد التالية: $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$ و $\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$ و $\cos(x - \pi) = \cos(x + \pi) = -\cos(x)$ و $\sin(x - \pi) = \sin(x + \pi) = -\sin(x)$	
تمرين 5 : x عدد حقيقي	
1 $\sin^4 x - \cos^4 x = (\sin^2 x)^2 - (\cos^2 x)^2 = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 1 \times (\sin^2 x - \cos^2 x)$ $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$	
2 $\sin^4 x + \cos^4 x = \sin^4 x + 2\sin^2 x \cos^2 x + \cos^4 x - 2\sin^2 x \cos^2 x$ $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x = 1 - 2\sin^2 x \cos^2 x$	
3 $\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3 = (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^2 x)$ $\sin^6 x + \cos^6 x = 1 \times (\sin^4 x + 2\sin^2 x \cos^2 x + \cos^2 x - 3\sin^2 x \cos^2 x)$ $\sin^6 x + \cos^6 x = (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x \cos^2 x$ $\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x$	
4 $\sin^2\left(\frac{2\pi}{5}\right) - \cos^2\left(\frac{2\pi}{5}\right) = \cos^2\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) - \sin^2\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) = \cos^2\left(\frac{\pi}{10}\right) - \sin^2\left(\frac{\pi}{10}\right)$	