



$$M = [a - (b - c - 1)] - [(-a - b) - (a - 1 + c)] - (b - c)$$

$$M = (a - b + c + 1) - (-a - b - a + 1 - c) - b + c$$

$$M = a - b + c + 1 + a + b + a - 1 + c - b + c = 3a - b + 3c$$

تمرين 3: أحسب وبسط: $A = \left(-1 + \frac{2}{5} + \frac{1}{4}\right)(-4) + \left(-\frac{3}{4} + \frac{5}{3} - \frac{2}{5}\right)\left(\frac{18}{5}\right)$

$$B = \left(\frac{4}{9} - \frac{11}{27}\right)\left(2 - \frac{4}{3}\right) - \left(\frac{3}{5} - \frac{7}{15}\right)\left(\frac{4}{3} - \frac{1}{2}\right)$$

$$C = \frac{7}{3}\left(\frac{3}{5} - \frac{2}{3} + \frac{3}{4}\right) + \left(-\frac{5}{6} + \frac{2}{3}\right)\left(\frac{1}{6} - \frac{2}{3}\right)$$

$$E = \frac{9 - \frac{1}{3} + \frac{5}{6}}{-5 + \frac{1}{2} - \frac{3}{4}} \times \frac{8 - \frac{1}{5} - \frac{7}{10}}{1 - \frac{3}{2} - \frac{5}{4}} \quad \text{و} \quad D = \frac{\frac{1}{2} - \frac{1}{3} + \frac{2}{5} + \frac{1}{6}}{\frac{1}{2} + \frac{1}{3} - \frac{2}{5} - \frac{5}{6}} + 1$$

$$G = \frac{1 - \frac{1}{3} + \frac{1}{1 + \frac{1}{3}}}{1 + \frac{1}{3} - \frac{1}{1 - \frac{1}{3}}} \quad \text{و} \quad F = 5 + \frac{1}{4 + \frac{1}{3 + \frac{1}{2}}}$$

الجواب: $A = \left(-1 + \frac{2}{5} + \frac{1}{4}\right)(-4) + \left(-\frac{3}{4} + \frac{5}{3} - \frac{2}{5}\right)\left(\frac{18}{5}\right)$

$$A = \left(\frac{-20 + 8 + 4}{20}\right)(-4) + \left(\frac{-27 + 60 - 8}{36}\right)\left(\frac{18}{5}\right)$$

$$A = \left(\frac{-7}{20}\right) \times (-4) + \frac{25}{36} \times \frac{18}{5} = \frac{7}{5} + \frac{5}{2} = \frac{14 + 25}{10} = \frac{39}{10}$$

$$B = \left(\frac{4}{9} - \frac{11}{27}\right)\left(2 - \frac{4}{3}\right) - \left(\frac{3}{5} - \frac{7}{15}\right)\left(\frac{4}{3} - \frac{1}{2}\right)$$

$$B = \frac{12 - 11}{27} \times \frac{6 - 4}{3} - \frac{9 - 7}{15} \times \frac{8 - 3}{6} = \frac{1}{27} \times \frac{2}{3} - \frac{2}{15} \times \frac{5}{6}$$

$$B = \frac{2}{81} - \frac{1}{9} = \frac{2 - 9}{81} = \frac{-7}{81}$$

$$C = \frac{7}{3}\left(\frac{3}{5} - \frac{2}{3} + \frac{3}{4}\right) + \left(-\frac{5}{6} + \frac{2}{3}\right)\left(\frac{1}{6} - \frac{2}{3}\right)$$

$$C = \frac{7}{3} \times \frac{36 - 40 + 45}{60} + \frac{-5 + 4}{6} \times \frac{1 - 4}{6} = \frac{7}{3} \times \frac{41}{60} + \frac{1}{6} \times \frac{3}{6}$$

$$C = \frac{287}{180} + \frac{1}{12} = \frac{287 + 15}{180} = \frac{302}{180} = \frac{151}{90}$$

$$D = \frac{\frac{1}{2} - \frac{1}{3} + \frac{2}{5} + \frac{1}{6}}{\frac{1}{2} + \frac{1}{3} - \frac{2}{5} - \frac{5}{6}} + 1 = \frac{\frac{15 - 10 + 12 + 5}{30}}{\frac{15 + 10 - 12 - 5}{30}} + 1 = \frac{22}{-12} + 1 = -\frac{22}{12} + \frac{12}{12}$$

$$D = \frac{-22 + 12}{12} = \frac{-10}{12} = \frac{-5}{6}$$

$$E = \frac{9 - \frac{1}{3} + \frac{5}{6}}{-5 + \frac{1}{2} - \frac{3}{4}} \times \frac{8 - \frac{1}{5} - \frac{7}{10}}{1 - \frac{3}{2} - \frac{5}{4}} = \frac{54 - 2 + 5}{-20 + 2 - 3} \times \frac{80 - 2 - 7}{4 - 6 - 5} = \frac{57}{-21} \times \frac{71}{-7}$$

$$E = \frac{57}{6} \times \frac{-4}{21} \times \frac{71}{10} \times \frac{-4}{7} = \frac{19}{3} \times \frac{2}{7} \times \frac{71}{5} \times \frac{2}{7} = \frac{5395}{735}$$

تمرين 1: باستعمال الرموز: $\in$; $\notin$; $\subset$; $\supset$ املأ الفراغات التالية: $6 \dots \mathbb{Z}$ و

$$-\frac{2}{3} \dots \mathbb{R}^+ \quad \text{و} \quad \mathbb{N} \dots \mathbb{Q} \quad \text{و} \quad \mathbb{Q} \dots \mathbb{R} \quad \text{و} \quad \sqrt{2} \dots \mathbb{R} \quad \text{و} \quad \sqrt{2} \dots \mathbb{Q} \quad \text{و} \quad \frac{2}{3} \dots \mathbb{Q}$$

$$\text{و} \quad \frac{2}{3} \dots \mathbb{N} \quad \text{و} \quad \frac{6}{2} \dots \mathbb{N} \quad \text{و} \quad \frac{\sqrt{100}}{5} \dots \mathbb{N} \quad \text{و} \quad \mathbb{Q} \dots \mathbb{Z} \quad \text{و} \quad \mathbb{Z} \dots \mathbb{Q} \quad \text{و} \quad \pi \dots \mathbb{Z}$$

$$\text{و} \quad 0 \dots \mathbb{Q}^* \quad \text{و} \quad -\frac{7}{3} \dots \mathbb{Q}^{*+} \quad \text{و} \quad \sqrt{16} \dots \mathbb{N} \quad \text{و} \quad 0 \dots \mathbb{R}^*$$

$$\mathbb{R}^* \dots \mathbb{R} \quad \text{و} \quad \mathbb{R}^+ \dots \mathbb{R} \quad \text{و} \quad \mathbb{Q} \dots \mathbb{R}^+ \quad \text{و} \quad \mathbb{Q}^* \dots \mathbb{R} \quad \text{و} \quad \{1; 3; -8\} \dots \mathbb{N} \quad \text{و} \quad \mathbb{R}^{**} \dots \mathbb{R}$$

الجواب: $6 \in \mathbb{Z}$ و $\frac{2}{3} \in \mathbb{Q}$ و $\sqrt{2} \notin \mathbb{Q}$ و $\sqrt{2} \in \mathbb{R}$ و $\mathbb{Q} \subset \mathbb{R}$

$$\mathbb{N} \subset \mathbb{Q} \quad \text{و} \quad -\frac{2}{3} \notin \mathbb{R}^+ \quad \text{و} \quad \frac{2}{3} \notin \mathbb{N} \quad \text{و} \quad \frac{6}{2} \in \mathbb{N} \quad \text{و} \quad \frac{\sqrt{100}}{5} \in \mathbb{N}$$

$$\mathbb{Q} \not\subset \mathbb{Z} \quad \text{و} \quad \mathbb{Z} \subset \mathbb{Q} \quad \text{و} \quad \pi \notin \mathbb{Z} \quad \text{و} \quad 0 \notin \mathbb{Q}^* \quad \text{و} \quad -\frac{7}{3} \notin \mathbb{Q}^{*+}$$

$$\sqrt{16} \in \mathbb{N} \quad \text{و} \quad 0 \notin \mathbb{R}^* \quad \text{و} \quad \mathbb{R}^* \subset \mathbb{R} \quad \text{و} \quad \mathbb{R}^+ \subset \mathbb{R} \quad \text{و} \quad \mathbb{Q} \not\subset \mathbb{R}^+$$

$$\mathbb{Q}^* \subset \mathbb{R} \quad \text{و} \quad \{1; 3; -8\} \not\subset \mathbb{N} \quad \text{و} \quad \mathbb{R}^{**} \subset \mathbb{R}$$

تمرين 2: أحسب وبسط: $A = \frac{3}{4} + \frac{5}{3} - \frac{7}{6}$ و $B = \frac{-2}{3} + \frac{7}{6} - \frac{1}{4} - 2$

$$C = \left(\frac{2}{3} - \frac{5}{2}\right)^2 \quad \text{و} \quad D = \frac{5 + \frac{1}{3}}{2 - \frac{3}{2}} \quad \text{و} \quad E = \left(1 - \frac{1}{3}\right)\left(\frac{2}{5} + 1 - \frac{1}{2}\right)$$

$$F = \frac{7 - \frac{4}{\pi}}{12 - 21\pi} \quad \text{و} \quad G = [(a - c) - (a - b)] - [(c - a) + (b - c)]$$

$$M = [a - (b - c - 1)] - [(-a - b) - (a - 1 + c)] - (b - c)$$

الجواب: $A = \frac{3}{4} + \frac{5}{3} - \frac{7}{6} = \frac{9}{12} + \frac{20}{12} - \frac{14}{12} = \frac{9 + 20 - 14}{12} = \frac{15}{12} = \frac{5}{4}$

$$B = \frac{-2}{3} + \frac{7}{6} - \frac{1}{4} - 2 = \frac{-8}{12} + \frac{14}{12} - \frac{3}{12} - \frac{24}{12} = \frac{-8 + 14 - 3 - 24}{12} = \frac{-21}{12} = -\frac{7}{4}$$

$$C = \left(\frac{2}{3} - \frac{5}{2}\right)^2 = \left(\frac{4 - 15}{6}\right)^2 = \left(\frac{-11}{6}\right)^2 = \frac{(-11)^2}{6^2} = \frac{121}{36}$$

$$D = \frac{5 + \frac{1}{3}}{2 - \frac{3}{2}} = \frac{\frac{16}{3}}{\frac{1}{2}} = \frac{16}{3} \times \frac{2}{1} = \frac{32}{3}$$

$$E = \left(1 - \frac{1}{3}\right)\left(\frac{2}{5} + 1 - \frac{1}{2}\right) = \left(\frac{3}{3} - \frac{1}{3}\right)\left(\frac{4}{10} + \frac{10}{10} - \frac{5}{10}\right) = \frac{2}{3}\left(\frac{4 + 10 - 5}{10}\right) = \frac{2}{3}\left(\frac{9}{10}\right) = \frac{2}{3} \times \frac{3 \times 3}{5 \times 2} = \frac{3}{5}$$

$$F = \frac{7 - \frac{4}{\pi}}{12 - 21\pi} = \frac{\frac{7\pi - 4}{\pi}}{\frac{7\pi - 4}{\pi}} = \frac{7\pi - 4}{\pi} \times \frac{1}{12 - 21\pi} = \frac{7\pi - 4}{\pi} \times \frac{1}{12 - 21\pi}$$

$$F = \frac{7\pi - 4}{\pi} \times \frac{1}{-3(7\pi - 4)} = -\frac{1}{3\pi}$$

$$G = [(a - c) - (a - b)] - [(c - a) + (b - c)] = (a - c - a + b) - (c - a + b - c)$$

$$G = a - c - a + b - c + a - b + c = a - c$$

$$E = \frac{3+2\sqrt{15}+5-(3-2\sqrt{15}+5)}{(\sqrt{3})^2-(\sqrt{5})^2} = \frac{3+2\sqrt{15}+5-3+2\sqrt{15}-5}{(\sqrt{3})^2-(\sqrt{5})^2} = \frac{4\sqrt{15}}{-2} = -2\sqrt{15}$$

$$F = (-\sqrt{10}) \times (-\sqrt{10}) + \sqrt{5} \times (-\sqrt{20}) - \sqrt{2} \times \sqrt{32}$$

$$F = (-\sqrt{10})^2 - \sqrt{5} \times \sqrt{5 \times 4} - \sqrt{2} \times \sqrt{2 \times 16} = 10 - \sqrt{5}^2 \times 2 - \sqrt{2}^2 \times 4$$

$$F = 10 - 10 - 8 = -8$$

$$G = \sqrt{3} \times 2\sqrt{3} - 8\sqrt{8} \times 2\sqrt{2} - 3\sqrt{5} \times \sqrt{20} + \sqrt{2} \times \sqrt{3} \times \sqrt{24}$$

$$G = 2 \times \sqrt{3}^2 - 8\sqrt{8} \times \sqrt{8} - 3\sqrt{5} \times \sqrt{5 \times 4} + \sqrt{2} \times \sqrt{3} \times \sqrt{2 \times 2 \times 3}$$

$$G = 2 \times 3 - 8\sqrt{8}^2 - 3\sqrt{5}^2 \times 2 + \sqrt{2}^2 \times \sqrt{3}^2 \times 2$$

$$G = 6 - 64 - 30 + 12 = -76$$

$$H = 5\sqrt{12} + 8\sqrt{27} + \sqrt{75} - 2\sqrt{48} - \sqrt{147}$$

$$H = 5\sqrt{4 \times 3} + 8\sqrt{9 \times 3} + \sqrt{25 \times 3} - 2\sqrt{16 \times 3} - \sqrt{49 \times 3}$$

$$H = 10\sqrt{3} + 24\sqrt{3} + 5\sqrt{3} - 8\sqrt{3} - 7\sqrt{3} = 24\sqrt{3}$$

تمرين 6: بين أن العدد $E = \frac{5\sqrt{7}}{\sqrt{2}-\sqrt{7}} + \frac{5\sqrt{2}}{\sqrt{2}+\sqrt{7}}$ عدد صحيح نسبي

الجواب:

$$E = \frac{5\sqrt{7}}{\sqrt{2}-\sqrt{7}} + \frac{5\sqrt{2}}{\sqrt{2}+\sqrt{7}} = \frac{(5\sqrt{7})(\sqrt{2}+\sqrt{7}) + 5\sqrt{2}(\sqrt{2}-\sqrt{7})}{(\sqrt{2}-\sqrt{7})(\sqrt{2}+\sqrt{7})}$$

$$E = \frac{5\sqrt{7}\sqrt{2} + 5\sqrt{7}\sqrt{7} + 5\sqrt{2}\sqrt{2} - 5\sqrt{2}\sqrt{7}}{(\sqrt{2})^2 - (\sqrt{7})^2} = \frac{35 + 10}{(\sqrt{2})^2 - (\sqrt{7})^2} = \frac{45}{-5} = -9 \in \mathbb{Z}$$

تمرين 7: اجعل مقامات الأعداد التالية جذرية :

$$C = \frac{\sqrt{5}+3\sqrt{7}}{2\sqrt{5}-4\sqrt{7}} \quad \text{و} \quad B = \frac{2-\sqrt{3}}{\sqrt{2}-3} \quad \text{و} \quad A = \frac{2+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

الجواب:

$$A = \frac{2+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(2+\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} = \frac{(2+\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2})^2 - (\sqrt{3})^2}$$

$$A = \frac{2\sqrt{2} + 2\sqrt{3} + \sqrt{6} + (\sqrt{3})^2}{2-3} = \frac{2\sqrt{2} + 2\sqrt{3} + \sqrt{6} + 3}{-1}$$

$$A = -(2\sqrt{2} + 2\sqrt{3} + \sqrt{6} + 3)$$

$$B = \frac{2-\sqrt{3}}{\sqrt{2}-3} = \frac{(2-\sqrt{3})(\sqrt{2}+3)}{(\sqrt{2}-3)(\sqrt{2}+3)} = \frac{2\sqrt{2} + 6 - \sqrt{6} - 3\sqrt{3}}{(\sqrt{2})^2 - (3)^2}$$

$$B = \frac{2\sqrt{2} - \sqrt{6} - 3\sqrt{3} + 6}{2-9} = \frac{\sqrt{6} - 2\sqrt{2} + 3\sqrt{3} - 6}{7}$$

$$C = \frac{\sqrt{5}+3\sqrt{7}}{2\sqrt{5}-4\sqrt{7}} = \frac{(\sqrt{5}+3\sqrt{7})(2\sqrt{5}+4\sqrt{7})}{(2\sqrt{5}-4\sqrt{7})(2\sqrt{5}+4\sqrt{7})}$$

$$C = \frac{\sqrt{5}+3\sqrt{7}}{2\sqrt{5}-4\sqrt{7}} = \frac{10+4\sqrt{35}+6\sqrt{35}+84}{(2\sqrt{5})^2 - (4\sqrt{7})^2} = \frac{94+10\sqrt{35}}{20-112}$$

$$C = -\frac{47+5\sqrt{35}}{46}$$

تمرين 8: نعتبر الأعداد التالية :

$$c = \frac{4}{\sqrt{3}-\sqrt{11}} \quad \text{و} \quad b = \frac{2}{\sqrt{7}-\sqrt{3}} \quad \text{و} \quad a = \frac{2}{\sqrt{11}-\sqrt{7}}$$

(1) اجعل مقامات الأعداد جذرية

(2) بسط العدد :

$$A = \frac{2}{\sqrt{11}-\sqrt{7}} + \frac{2}{\sqrt{7}-\sqrt{3}} + \frac{4}{\sqrt{3}-\sqrt{11}}$$

الجواب:

$$a = \frac{2}{\sqrt{11}-\sqrt{7}} = \frac{2(\sqrt{11}+\sqrt{7})}{(\sqrt{11}-\sqrt{7})(\sqrt{11}+\sqrt{7})} = \frac{2(\sqrt{11}+\sqrt{7})}{(\sqrt{11})^2 - (\sqrt{7})^2}$$

$$a = \frac{2(\sqrt{11}+\sqrt{7})}{11-7} = \frac{2(\sqrt{11}+\sqrt{7})}{4} = \frac{\sqrt{11}+\sqrt{7}}{2}$$

$$F = 5 + \frac{1}{4 + \frac{1}{3 + \frac{1}{2}}} = 5 + \frac{1}{4 + \frac{2}{7}} = 5 + \frac{7}{30} = \frac{150+7}{30} = \frac{157}{30}$$

$$G = \frac{1 - \frac{1}{3} + \frac{1}{1 + \frac{1}{3}}}{1 + \frac{1}{3} - \frac{1}{1 - \frac{1}{3}}} = \frac{\frac{3-1}{3} + \frac{3}{3+1}}{\frac{3+1}{3} - \frac{3}{3-1}} = \frac{\frac{2}{3} + \frac{3}{4}}{\frac{4}{3} - \frac{3}{2}} = \frac{\frac{17}{12}}{-\frac{1}{6}} = \frac{17}{12} \times -6 = -\frac{17}{2}$$

تمرين 4: ليكن a و b عددين حقيقيين بحيث : $a-b = -\frac{7}{6}$

أحسب ما يلي : $A_1 = a - \left(b - \frac{71}{61}\right)$ و $A_2 = \left(a - \frac{9}{5}\right) - \left(b - \frac{9}{5}\right)$

$$A_4 = (2a-5) + (6-2b) \quad \text{و} \quad A_3 = \left(b + \frac{2016}{2017}\right) - \left(a - \frac{1}{2017}\right)$$

الجواب:

$$A_1 = a - \left(b - \frac{71}{61}\right) = a - b + \frac{71}{61} = -\frac{7}{6} + \frac{71}{61} = -\frac{853}{336}$$

$$A_2 = \left(a - \frac{9}{5}\right) - \left(b - \frac{9}{5}\right) = a - \frac{9}{5} - b + \frac{9}{5} = a - b = -\frac{7}{6}$$

$$A_3 = \left(b + \frac{2016}{2017}\right) - \left(a - \frac{1}{2017}\right) = b - a + \frac{2016}{2017} + \frac{1}{2017}$$

$$A_3 = -(a-b) + \frac{2016+1}{2017} = -\left(-\frac{7}{6}\right) + \frac{2017}{2017} = \frac{7}{6} + 1 = \frac{7}{6} + \frac{6}{6} = \frac{13}{6}$$

$$A_4 = (2a-5) + (6-2b) = 2a-5+6-2b = 2a-2b+1$$

$$A_4 = 2(a-b) + 1 = 2 \times -\frac{7}{6} + 1 = -\frac{7}{3} + 1 = -\frac{4}{3}$$

تمرين 5: بسط $A = \sqrt{\frac{9}{2}}$ و $B = \frac{\sqrt{28}}{\sqrt{14}}$

$$C = 3\sqrt{20} + 4\sqrt{45} - 2\sqrt{80} - \sqrt{180}$$

$$E = \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} - \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}} \quad \text{و} \quad D = (\sqrt{3}+\sqrt{2}-\sqrt{5})(\sqrt{3}+\sqrt{2}+\sqrt{5})$$

$$F = (-\sqrt{10}) \times (-\sqrt{10}) + \sqrt{5} \times (-\sqrt{20}) - \sqrt{2} \times \sqrt{32}$$

$$G = \sqrt{3} \times 2\sqrt{3} - 8\sqrt{8} \times 2\sqrt{2} - 3\sqrt{5} \times \sqrt{20} + \sqrt{2} \times \sqrt{3} \times \sqrt{24}$$

$$H = 5\sqrt{12} + 8\sqrt{27} + \sqrt{75} - 2\sqrt{48} - \sqrt{147}$$

الجواب:

$$A = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{(\sqrt{2})^2} = \frac{3\sqrt{2}}{2}$$

$$B = \frac{\sqrt{28}}{\sqrt{14}} = \sqrt{\frac{28}{14}} = \sqrt{2}$$

$$C = 3\sqrt{20} + 4\sqrt{45} - 2\sqrt{80} - \sqrt{180} = 3\sqrt{4 \times 5} + 4\sqrt{9 \times 5} - 2\sqrt{16 \times 5} - \sqrt{36 \times 5}$$

$$C = 3 \times 2\sqrt{5} + 4 \times 3\sqrt{5} - 2 \times 4\sqrt{5} - 6\sqrt{5} = 6\sqrt{5} + 12\sqrt{5} - 8\sqrt{5} - 6\sqrt{5} = (6+12-8-6)\sqrt{5}$$

$$C = 4\sqrt{5}$$

$$D = (\sqrt{3}+\sqrt{2}-\sqrt{5})(\sqrt{3}+\sqrt{2}+\sqrt{5}) = ((\sqrt{3}+\sqrt{2})-\sqrt{5})((\sqrt{3}+\sqrt{2})+\sqrt{5})$$

$$D = (\sqrt{3}+\sqrt{2})^2 - (\sqrt{5})^2 = (\sqrt{3})^2 + 2\sqrt{3}\sqrt{2} + (\sqrt{2})^2 - 5 = 3 + 2\sqrt{3 \times 2} + 2 - 5$$

$$D = 2\sqrt{6}$$

$$E = \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} - \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{(\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5}) - (\sqrt{3}-\sqrt{5})(\sqrt{3}-\sqrt{5})}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})}$$

$$E = \frac{(\sqrt{3}+\sqrt{5})^2 - (\sqrt{3}-\sqrt{5})^2}{(\sqrt{3})^2 - (\sqrt{5})^2} = \frac{(\sqrt{3})^2 + 2\sqrt{3}\sqrt{5} + (\sqrt{5})^2 - ((\sqrt{3})^2 - 2\sqrt{3}\sqrt{5} + (\sqrt{5})^2)}{(\sqrt{3})^2 - (\sqrt{5})^2}$$

• $3,25 \times 10^4$ هي كتابة علمية صحيح

• 15×10^3 هي كتابة علمية خطأ

• -1.7×10^7 هي كتابة علمية صحيح

تمرين 13: حدد الكتابة العلمية للأعداد التالية

300 000 و 45 000 0 و 0,01 و 0,001

و 368 100 000 و 25 000 000 و 0,0002

الجواب: $450000 = 4.5 \times 10^5$ و $300000 = 3 \times 10^5$ و $0.01 = 10^{-2}$

و $368100000 = 3.681 \times 10^9$ و $0.001 = 10^{-3}$

$0.0002 = 2 \times 10^{-4}$ و $25000000 = 2.5 \times 10^7$

تمرين 14: بسط أو أكتب على شكل قوى :

$$B = (-3)^1 \times (-3)^5 \times (3)^2 \times (-3)^{-10} \quad A = 2^3 \times (2^2)^4 \times (2^{-5})^3$$

$$D = \frac{(-2)^3 \times (4^2)^{-1} \times 8}{1024 \times (-16)^{-4}} \quad C = \frac{3^{-5} \times 4^{-2} \times 9}{12^3} \times \frac{9}{2^2}$$

$$F = \frac{3 \times 10^{-5} \times 7,2 \times 10^7}{2 \times 15^3} \quad \text{و} \quad E = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5}$$

الجواب: $A = 2^3 \times (2^2)^4 \times (2^{-5})^3 = 2^3 \times 2^{2 \times 4} \times 2^{-5 \times 3} = 2^{3+8-15} = 2^{-4}$

$$A = \frac{1}{2^4} = \frac{1}{16}$$

$$B = (-3)^1 \times (-3)^5 \times (3)^2 \times (-3)^{-10} = -3^1 \times -3^5 \times 3^2 \times 3^{-10}$$

$$B = 3^1 \times 3^5 \times 3^2 \times 3^{-10} = 3^{1+5+2-10} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$C = \frac{3^{-5} \times 4^{-2} \times 9}{12^3} \times \frac{9}{2^2} = \frac{3^{-5} \times (2^2)^{-2} \times 3^2}{(3 \times 2^2)^3} \times \frac{3^2}{2^2} = \frac{3^{-5} \times 2^{-4} \times 3^2}{(3)^3 \times 2^6 \times 2^2}$$

$$C = \frac{3^{-5} \times (2)^{-4} \times 3^2}{(3)^3 \times 2^6 \times 2^2} = 3^{-5} \times 2^{-4} \times 3^2 \times (3)^{-3} \times 2^{-6} \times 2^{-2} = 3^{-5+2-3} \times 2^{-4-6-2} = 3^{-6} \times 2^{-12}$$

$$C = 3^{-6} \times 2^{-12}$$

$$D = \frac{(-2)^3 \times (4^2)^{-1} \times 8}{1024 \times (-16)^{-4}} = \frac{-2^3 \times 4^{2 \times (-1)} \times 2^3}{1024 \times (-2^3)^{-4}} = \frac{-2^3 \times (2^2)^{-2} \times 2^3}{2^{10} \times (-2^3)^{-4}}$$

$$D = -2^3 \times (2^2)^{-2} \times 2^3 \times 2^{-10} \times (-2)^{3 \times 4} = -2^{3-4+3-10+12} = -2^4 = -16$$

$$E = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5} = 10^{-8} \times 10^9 \times 10^7 \times 10^{-4} \times 10^2 \times 10^{-3} \times 10^{-5}$$

$$E = 10^{-8+9+7-4+2-3-5} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

$$F = \frac{3 \times 10^{-5} \times 7,2 \times 10^7}{2 \times 15^3} = \frac{3 \times 10^{-5} \times 3^2 \times 2^3 \times 10^{-1} \times 10^7}{2 \times (3 \times 5)^3}$$

$$F = \frac{3 \times 3^2 \times 2^3 \times 10}{2 \times 3^3 \times 5^3} = \frac{3 \times 3^2 \times 2^3 \times 2 \times 5}{2 \times 3^3 \times 5^3} = \frac{2^3}{5^2} = \frac{8}{25}$$

تمرين 15: أحسب وبسط: a عدد حقيقي غير منعدم

$$A = (\sqrt{2})^{-2} \times (\sqrt{2})^2 \times (-\sqrt{2})^{-5} \times (\sqrt{2})^3$$

$$C = \left(\left(-\frac{3}{2} \right)^{-1} \right)^4 \quad \text{و} \quad B = \left((-\sqrt{3})^{-2} \right)^2$$

$$E = \left(\frac{a \times (a^{-3})^{-2}}{a^{-2} \times (a^4 \times a^7)^2} \right)^{-3} \quad \text{و} \quad D = \frac{a^{-2} \times (-a)^5}{-a \times a^{-4}} \times \frac{a^{-1} \times (a^{-2})^5}{((-a)^4)^{-2}}$$

$$G = \left(\frac{5^3 \times 2^{-3}}{4 \times 25} \right)^2 \times \frac{2^8}{10^2 \times 5} \quad \text{و} \quad F = \left(-\frac{1}{8} \right)^2 \times \left(\frac{2}{5} \right)^6 \times \left(-\frac{5}{2} \right)^3$$

الجواب: $A = (\sqrt{2})^{-2} \times (\sqrt{2})^2 \times (-\sqrt{2})^{-5} \times (\sqrt{2})^3$

$$b = \frac{2}{\sqrt{7}-\sqrt{3}} = \frac{2(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} = \frac{2(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2-(\sqrt{3})^2}$$

$$b = \frac{2(\sqrt{7}+\sqrt{3})}{7-3} = \frac{2(\sqrt{7}+\sqrt{3})}{4} = \frac{\sqrt{7}+\sqrt{3}}{2}$$

$$c = \frac{4}{\sqrt{3}-\sqrt{11}} = \frac{4(\sqrt{3}+\sqrt{11})}{(\sqrt{3}-\sqrt{11})(\sqrt{3}+\sqrt{11})} = \frac{2(\sqrt{3}+\sqrt{11})}{(\sqrt{3})^2-(\sqrt{11})^2}$$

$$c = \frac{2(\sqrt{3}+\sqrt{11})}{3-11} = \frac{2(\sqrt{3}+\sqrt{11})}{-8} = -\frac{\sqrt{3}+\sqrt{11}}{4}$$

(2) بسط العدد : A

$$A = \frac{\sqrt{11}+\sqrt{7}}{2} + \frac{\sqrt{7}+\sqrt{3}}{2} - \frac{\sqrt{7}+\sqrt{11}}{2} \text{ يعني } A = a+b+c$$

$$A = \frac{\sqrt{11}+\sqrt{7}+\sqrt{7}+\sqrt{3}-\sqrt{3}-\sqrt{11}}{2} = \frac{2\sqrt{7}}{2} = \sqrt{7}$$

تمرين 9: (1) أكتب العدد : $\frac{\sqrt{2}+1}{\sqrt{2}-1}$ على شكل كسر مقامه عدد جذري

(2) استنتج تبسيطا للعدد : $G = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$ بسط العدد : (3) $H = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$

الجواب: (1)

$$\frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{(\sqrt{2}+\sqrt{1})^2}{(\sqrt{2}-\sqrt{1})(\sqrt{2}+\sqrt{1})} = \frac{3+2\sqrt{2}}{(\sqrt{2})^2-(\sqrt{1})^2} = \frac{3+2\sqrt{2}}{2-1} = 3+2\sqrt{2}$$

$$G = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \sqrt{3+2\sqrt{2}} = \sqrt{(\sqrt{2}+1)^2} = |\sqrt{2}+1| = \sqrt{2}+1 \quad (2)$$

$$H = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \frac{1}{G} = \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1 \quad (3)$$

تمرين 10: أحسب وبسط $A = \sqrt{2-\sqrt{2+\sqrt{2}}} \times \sqrt{2+\sqrt{2+\sqrt{2}}} \times \sqrt{2+\sqrt{2}} \times \sqrt{2}$

الجواب: $A = \sqrt{(2-\sqrt{2+\sqrt{2}})(2+\sqrt{2+\sqrt{2}})} \times \sqrt{2+\sqrt{2}} \times \sqrt{2}$

$$A = \sqrt{2^2 - (\sqrt{2+\sqrt{2}})^2} \times \sqrt{2+\sqrt{2}} \times \sqrt{2} = \sqrt{4 - (2+\sqrt{2})} \times \sqrt{2+\sqrt{2}} \times \sqrt{2}$$

$$A = \sqrt{2-\sqrt{2}} \times \sqrt{2+\sqrt{2}} \times \sqrt{2} = \sqrt{2^2 - (\sqrt{2})^2} \times \sqrt{2} = \sqrt{2} \times \sqrt{2} = 2$$

تمرين 11: أحسب وبسط: $A = \frac{(10^6)^4 \times 10^{-2}}{10^4 \times 10^6 \times 10^{-13}}$ و

$$B = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5}$$

الجواب: $A = \frac{(10^6)^4 \times 10^{-2}}{10^4 \times 10^6 \times 10^{-13}} = 10^{24} \times 10^{-2} \times 10^{-4} \times 10^{-6} \times 10^{13} = 10^{-1} = \frac{1}{10}$

$$B = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5} = 10^{-8} \times 10^9 \times 10^7 \times 10^{-4} \times 10^2 \times 10^{-3} \times 10^{-5}$$

$$B = 10^{-8+9+7-4+2-3-5} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

تمرين 12: أجب بصحيح أو خطأ

• الكتابة العلمية للعدد : 149597870 كلم

هي $1,4959787 \times 10^8$ كلم.

• $3,25 \times 10^4$ هي كتابة علمية

• 15×10^3 هي كتابة علمية

• الكتابة العلمية للعدد : -17000000 هي -1.7×10^7

الجواب: الكتابة العلمية تكون على الشكل $x = a \times 10^p$ أو $x = -a \times 10^p$

حيث p ينتمي الى $\mathbb{Z}$ و a عدد عشري بحيث $1 \leq a < 10$ ومنه

• $1,4959787 \times 10^8$ هي كتابة علمية صحيح

$$D = 27x^3 - 54x^2 + 36x - 8$$

$$E = (x+2)(x^2 - 2x + 4) = (x+2)(x^2 - 2 \times x + 2^2) = x^3 + 2^3 = x^3 + 8$$

$$F = (200520052006)^2 - (200520052005 \times 200520052007) : \text{حساب}$$

نلاحظ أن الأعداد الثلاثة تختلف فقط في رقم وحداتها لتبسيط الحساب

$$x = 200520052006 : \text{نضع}$$

$$200520052007 = x + 1 \text{ و } 200520052005 = x - 1 : \text{إن}$$

$$F = x^2 - (x-1)(x+1) : \text{ومن}$$

$$F = x^2 - (x^2 - 1) = x^2 - x^2 + 1 = 1$$

تمرين 18: أنشر وأحسب وبسط حيث $x \in \mathbb{R}$

$$B = (4\sqrt{3} - 7)^{2015} \times (4\sqrt{3} + 7)^{2015} \text{ و } A = (3 + \sqrt{11})^2 - (3 - \sqrt{11})^2$$

$$D = (5x + 2)^3 \quad C = (\sqrt{75} - \sqrt{98}) \times (5\sqrt{3} + 7\sqrt{2})$$

$$F = (2x - 3)(4x^2 + 6x + 9) \quad E = (\sqrt{3} - 1)^3$$

$$G = (2015200052004)^2 - (2015200052002 \times 2015200052006)$$

$$M = (x^2 - 2x + 1)^2 \text{ و } H = \left(\frac{x}{2} + 2\sqrt{3}\right)^2 + \left(x\sqrt{5} - \frac{3}{2}\right)^2$$

$$R = \left(x^3 + \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2} - x^3\right) \text{ و } N = (x\sqrt{2} + \sqrt{5})(\sqrt{5} - x\sqrt{2})$$

$$L = (3x + \sqrt{2} - \sqrt{5})(3x + \sqrt{2} + \sqrt{5})$$

الجواب:

$$A = (\sqrt{3} + \sqrt{11})^2 - (\sqrt{3} - \sqrt{11})^2 = (\sqrt{3})^2 + 2\sqrt{3}\sqrt{11} + (\sqrt{11})^2 - ((\sqrt{3})^2 - 2\sqrt{3}\sqrt{11} + (\sqrt{11})^2)$$

$$A = 3 + 2\sqrt{33} + 11 - (3 - 2\sqrt{33} + 11) = 3 + 2\sqrt{33} + 11 - 3 + 2\sqrt{33} - 11 = 4\sqrt{33}$$

$$B = ((4\sqrt{3} - 7)(4\sqrt{3} + 7))^{2015} = ((4\sqrt{3})^2 - (7)^2)^{2015} = (48 - 49)^{2015} = (-1)^{2015} = -1$$

$$C = (\sqrt{75} - \sqrt{98}) \times (5\sqrt{3} + 7\sqrt{2}) = (\sqrt{25 \times 3} - \sqrt{49 \times 2}) \times (5\sqrt{3} + 7\sqrt{2})$$

$$C = (5\sqrt{3} - 7\sqrt{2}) \times (5\sqrt{3} + 7\sqrt{2}) = (5\sqrt{3})^2 - (7\sqrt{2})^2 = 75 - 98 = 75 - 98 = -23$$

$$D = (5x + 2)^3 = (5x)^3 + 3(5x)^2 \times 2 + 3 \times 5x \times (2)^2 + (2)^3$$

$$D = 125x^3 + 150x^2 + 60x + 8$$

$$E = (\sqrt{3} - 1)^3 = (\sqrt{3})^3 - 3(\sqrt{3})^2 \times 1 + 3 \times \sqrt{3} \times (1)^2 - (1)^3$$

$$E = 3\sqrt{3} - 9 + 3 \times \sqrt{3} - 1 = 6\sqrt{3} - 10$$

$$F = (2x - 3)(4x^2 + 6x + 9) = (2x - 3)((2x)^2 + 2x \times 3 + 3^2) = (2x)^3 - 3^3 = 8x^3 - 27$$

$$G = (2015200052004)^2 - (2015200052002 \times 2015200052006) : \text{حساب}$$

نلاحظ أن الأعداد الثلاثة تختلف فقط في رقم وحداتها لتبسيط الحساب

$$x = 2015200052004 : \text{نضع}$$

$$x + 2 = 2015200052006 \text{ و } x - 2 = 2015200052002 : \text{إن}$$

$$G = x^2 - (x - 2)(x + 2) : \text{ومن}$$

$$G = x^2 - (x^2 - 4) = x^2 - x^2 + 4 = 4$$

$$H = \left(\frac{x}{2} + 2\sqrt{3}\right)^2 + \left(x\sqrt{5} - \frac{3}{2}\right)^2 =$$

$$H = \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times 2\sqrt{3} + (2\sqrt{3})^2 + (x\sqrt{5})^2 - 2x\sqrt{5} \times \frac{3}{2} + \left(\frac{3}{2}\right)^2$$

$$H = \frac{x^2}{4} + 2x\sqrt{3} + 12 + 5x^2 - 3x\sqrt{5} + \frac{9}{4}$$

$$H = \frac{21}{4}x^2 + (2\sqrt{3} - 3\sqrt{5})x + \frac{57}{4}$$

$$M = (x^2 - 2x + 1)^2 = ((x^2 - 2x) + 1)^2$$

$$A = -(\sqrt{2})^{(-2+2-5+3)} = -(\sqrt{2})^{-2} = -\frac{1}{(\sqrt{2})^2} = -\frac{1}{2}$$

$$B = ((-\sqrt{3})^{-2})^2 = \left(\frac{1}{\sqrt{3}^2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$C = \left(\left(-\frac{3}{2}\right)^{-1}\right)^4 = \left(-\frac{3}{2}\right)^{-4} = \left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = 2^4 \times 3^{-4}$$

$$D = \frac{a^{-2} \times (-a)^5}{-a \times a^{-4}} \times \frac{a^{-1} \times (a^{-2})^5}{((-a)^4)^{-2}} = \frac{-a^{-2} \times a^5 \times a^{-1} \times a^{-10}}{-a \times a^{-4} \times a^{-8}}$$

$$D = \frac{a^{(-2+5-1-10)}}{a^{(-4-8)}} = \frac{a^{-8}}{a^{-11}} = a^{-8+11} = a^3$$

$$E = \left(\frac{a \times (a^{-3})^{-2}}{a^{-2} \times (a^{-4} \times a^7)^2}\right)^{-3} = \left(\frac{a \times a^6}{a^{-2} \times a^6}\right)^{-3} = \left(\frac{a}{a^{-2}}\right)^{-3} = (a \times a^2)^{-3}$$

$$E = (a^3)^{-3} = a^{-9}$$

$$F = \left(-\frac{1}{8}\right)^2 \times \left(\frac{2}{5}\right)^6 \times \left(-\frac{5}{2}\right)^3 = -\left(\frac{1}{2^3}\right)^2 \times \frac{2^6}{5^6} \times \frac{5^3}{2^3} = -\frac{2^6 \times 5^3}{2^6 \times 5^6 \times 2^3}$$

$$F = -\frac{2^6 \times 5^3}{2^9 \times 5^6} = -\frac{1}{2^3 \times 5^3} = -\frac{1}{(2 \times 5)^3} = -\frac{1}{10^3} = -10^{-3}$$

$$G = \left(\frac{5^3 \times 2^{-3}}{4 \times 25}\right)^2 \times \frac{2^8}{10^2 \times 5} = \frac{5^6 \times 2^{-6}}{(2^2 \times 5^2)^2} \times \frac{2^8}{(2^2 \times 5^2) \times 5}$$

$$G = \frac{5^6 \times 2^{-6} \times 2^8}{2^4 \times 5^4 \times 2^2 \times 5^2 \times 5} = \frac{5^6 \times 2^2}{2^6 \times 5^7} = \frac{1}{2^4 \times 5} = \frac{1}{80}$$

تمرين 16: a و b عدنان حقيقيان غير منعدمين

$$C = \frac{(ab^2)^3 \times a^4 b^2}{(ab)^5} : \text{نعتبر العدد الحقيقي}$$

(1) أحسب وبسط C (2) أكتب العدد C على شكل قوة أساسها 10 علما أن :

$$b = 100 \text{ و } a = \frac{1}{10}$$

الجواب: (1)

$$C = \frac{(ab^2)^3 \times a^4 b^2}{(ab)^5} = \frac{a^3 \times b^6 \times a^4 \times b^2}{a^5 \times b^5} = \frac{a^7 \times b^8}{a^5 \times b^5} = a^2 \times b^3$$

$$b = 100 = 10^2 \text{ و } a = \frac{1}{10} = 10^{-1} (2)$$

$$C = (10^{-1})^2 \times (10^2)^3 = 10^{-2} \times 10^6 = 10^4$$

تمرين 17: أحسب وبسط حيث $x \in \mathbb{R}$

$$B = [(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})]^2 \quad A = (\sqrt{5} + \sqrt{2})^2 - (\sqrt{5} - \sqrt{2})^2$$

$$E = (x+2)(x^2 - 2x + 4) \quad D = (3x - 2)^3 \quad C = (\sqrt{2} + 1)^3$$

$$F = (200520052006)^2 - (200520052005 \times 200520052007)$$

عندما تعجز الآلة الحاسبة

الجواب:

$$A = (\sqrt{5} + \sqrt{2})^2 - (\sqrt{5} - \sqrt{2})^2 = (\sqrt{5})^2 + 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2 - ((\sqrt{5})^2 - 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2)$$

$$A = 5 + 2\sqrt{10} + 2 - (5 - 2\sqrt{10} + 2) = 5 + 2\sqrt{10} + 2 - 5 + 2\sqrt{10} - 2 = 4\sqrt{10}$$

$$B = [(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})]^2 = ((\sqrt{2})^2 - (\sqrt{3})^2)^2 = (2 - 3)^2 = (-1)^2 = 1$$

$$C = (\sqrt{2} + 1)^3 = (\sqrt{2})^3 + 3(\sqrt{2})^2 \times 1 + 3\sqrt{2}(1)^2 + (1)^3 = 2\sqrt{2} + 3 \times 2 + 3\sqrt{2} + 1$$

$$C = 5\sqrt{2} + 7$$

$$D = (3x - 2)^3 = (3x)^3 - 3(3x)^2 \times 2 + 3 \times 3x \times (2)^2 - (2)^3$$

$$\left(\sqrt{a+\sqrt{a^2-b^2}}\right)^2 = \left(\frac{\sqrt{2}}{2}(\sqrt{a-b}+\sqrt{a+b})\right)^2 \text{ إذن وجدنا:}$$

$$\sqrt{a+\sqrt{a^2-b^2}} = \frac{\sqrt{2}}{2}(\sqrt{a-b}+\sqrt{a+b}) \text{ ومنه:}$$

تمرين 22: عمل التعبيرات التالية: $a \in \mathbb{R}$ و $b \in \mathbb{R}$ و $x \in \mathbb{R}$

$$B = 16 - 25x^2 \quad A = 16x^2 - 8x + 1$$

$$E = 27 + x^3 \quad D = (2x-1)^3 - 8 \quad C = 1 - (1-3x)^2$$

$$G = x^5 + x^3 - x^2 - 1 \quad F = x^{12} - 2x^6 + 1$$

$$M = x^4 - 49 \quad H = x^3 + 1 + 2(x^2 - 1) - (x+1)$$

$$N = a^2 + b^2 - x^2 + 2ab$$

$$L = 4x^2 - 4x\sqrt{5} + 5 + (1-2x)(2x-\sqrt{5})$$

$$R = x^2 - 6x + 8 \quad \text{و} \quad K = (x-2)(3x-4) + x^3 - 8$$



التمرين هو كتابة مجموع على شكل جذاء

$$A = 16x^2 - 8x + 1 = (4x)^2 - 2 \times 4x \times 1 + 1^2 = (4x-1)^2 \text{ **الجواب:**}$$

$$B = 16 - 25x^2 = (4)^2 - (5x)^2 = (4-5x)(4+5x)$$

$$C = 1 - (1-3x)^2 = 1^2 - (1-3x)^2 = (1-(1-3x))(1+(1-3x))$$

$$C = (1-1+3x)(1+1-3x) = 3x(2-3x)$$

$$D = (2x-1)^3 - 8 = (2x-1)^3 - 2^3 =$$

$$D = ((2x-1)-2)((2x-1)^2 + (2x-1) \times 2 + 2^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) \text{ حسب المتطابقة التالية:}$$

$$D = (2x-3)((2x)^2 - 4x + 1 + 4x - 2 + 4) = (2x-3)(4x^2 + 3)$$

$$E = 27 + x^3 = 3^3 + x^3 = (3+x)(3^2 - 3x + x^2)$$

$$E = (3+x)(9-3x+x^2)$$

$$F = (x^6)^2 - 2x^6 + 1 = (x^6)^2 - 2x^6 \times 1 + 1^2 = (x^6-1)^2$$

$$G = x^5 + x^3 - x^2 - 1 = x^3(x^2+1) - (x^2+1) = (x^3-1)(x^2+1)$$

$$H = x^3 + 1 + 2(x^2-1) - (x+1) = x^3 + 1^3 + 2(x^2-1^2) - (x+1)$$

$$H = (x+1)(x^2-x+1^2) + 2(x+1)(x-1) - (x+1)$$

$$H = (x+1)(x^2-x+1+2(x-1)-1) = (x+1)(x^2-x+1+2x-2-1)$$

$$H = (x+1)(x^2+x-2)$$

$$M = x^4 - 49 = x^4 - (\sqrt{7})^4 = (x^2)^2 - (\sqrt{7}^2)^2$$

$$M = (x^2 - \sqrt{7}^2)(x^2 + \sqrt{7}^2) = (x - \sqrt{7})(x + \sqrt{7})(x^2 + 7)$$

$$N = a^2 + b^2 - x^2 + 2ab = a^2 + 2ab + b^2 - x^2 = (a+b)^2 - x^2$$

$$N = (a+b-x)(a+b+x)$$

$$L = 4x^2 - 4x\sqrt{5} + 5 + (1-2x)(2x-\sqrt{5})$$

$$L = (2x)^2 - 2 \times 2x \sqrt{5} + (\sqrt{5})^2 + (1-2x)(2x-\sqrt{5})$$

$$L = (2x-\sqrt{5})^2 + (1-2x)(2x-\sqrt{5}) = (2x-\sqrt{5})(2x-\sqrt{5}+1-2x)$$

$$L = (2x-\sqrt{5})(1-\sqrt{5})$$

$$M = (x^2 - 2x)^2 + 2(x^2 - 2x) \times 1 + 1^2$$

$$M = (x^2)^2 - 2x^2 \times 2x + 4x^2 + 2x^2 - 4x + 1$$

$$M = x^4 - 4x^3 + 6x^2 - 4x + 1$$

$$N = (x\sqrt{2} + \sqrt{5})(\sqrt{5} - x\sqrt{2}) = (\sqrt{5} + x\sqrt{2})(\sqrt{5} - x\sqrt{2})$$

$$N = (\sqrt{5})^2 - (x\sqrt{2})^2 = 5 - 2x^2$$

$$R = \left(x^3 + \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2} - x^3\right) = \left(\frac{\sqrt{3}}{2} + x^3\right)\left(\frac{\sqrt{3}}{2} - x^3\right)$$

$$R = \left(\frac{\sqrt{3}}{2}\right)^2 - (x^3)^2 = \frac{3}{4} - x^6$$

$$L = (3x + \sqrt{2} - \sqrt{5})(3x + \sqrt{2} + \sqrt{5}) = ((3x + \sqrt{2}) - \sqrt{5})((3x + \sqrt{2}) + \sqrt{5})$$

$$L = (3x)^2 + 6x\sqrt{2} + 2 - 5 = 9x^2 + 6x\sqrt{2} - 3$$

تمرين 19: أتمم الفراغات التالية :

$$10 - 4\sqrt{6} = (\dots)^2 \quad \text{و} \quad 4 + 2\sqrt{2} = (\dots)^2$$

الجواب:

$$4 + 2\sqrt{3} = 4 + 2 \times \sqrt{3} \times 1 = 3 + 2 \times \sqrt{3} \times 1 + 1 = (\sqrt{3})^2 + 2 \times \sqrt{3} \times 1 + (1)^2$$

$$4 + 2\sqrt{3} = (\sqrt{3} + 1)^2$$

$$10 - 4\sqrt{6} = 10 - 2 \times 2 \times \sqrt{6} = (2)^2 + 2 \times \sqrt{6} \times 2 + (\sqrt{6})^2$$

$$10 - 4\sqrt{6} = (2 - \sqrt{6})^2$$

تمرين 20: أكتب التعبيرات التالية على شكل $(a-b)^2$ أو $(a+b)^2$:

$$3 - 2\sqrt{2} \quad (4 \quad 9 - 4\sqrt{5} \quad (3 \quad 6 + 4\sqrt{2} \quad (2 \quad 11 + 6\sqrt{2} \quad (1$$

$$7 - 4\sqrt{3} \quad (6 \quad 12 - 6\sqrt{3} \quad (5$$

الجواب:

$$11 + 6\sqrt{2} = 9 + 2 \times 3\sqrt{2} + 2 = 3^2 + 2 \times 3\sqrt{2} + (\sqrt{2})^2 = (3 + \sqrt{2})^2 \quad (1$$

$$6 + 4\sqrt{2} = 4 + 2 \times 2\sqrt{2} + 2 = 2^2 + 2 \times 2\sqrt{2} + (\sqrt{2})^2 = (2 + \sqrt{2})^2 \quad (2$$

$$9 - 4\sqrt{5} = 4 - 2 \times 2\sqrt{5} + 5 = 2^2 - 2 \times 2\sqrt{5} + (\sqrt{5})^2 = (2 - \sqrt{5})^2 \quad (3$$

$$3 - 2\sqrt{2} = 1 - 2 \times 1\sqrt{2} + 2 = 1^2 - 2 \times 1\sqrt{2} + (\sqrt{2})^2 = (1 - \sqrt{2})^2 \quad (4$$

$$12 - 6\sqrt{3} = 9 - 2 \times 3\sqrt{3} + 3 = 3^2 - 2 \times 3\sqrt{3} + (\sqrt{3})^2 = (3 - \sqrt{3})^2 \quad (5$$

$$7 - 4\sqrt{3} = 4 - 2 \times 2\sqrt{3} + 3 = 2^2 - 2 \times 2\sqrt{3} + (\sqrt{3})^2 = (2 - \sqrt{3})^2 \quad (6$$

تمرين 21: $a \geq b$ و $b \in \mathbb{R}^* \quad b \in \mathbb{R}^* \quad a \in \mathbb{R}^*$

$$\sqrt{a+\sqrt{a^2-b^2}} = \frac{\sqrt{2}}{2}(\sqrt{a-b}+\sqrt{a+b}) \text{ بين أن:}$$

الجواب:

لكي نبين أن عددين موجبين متساويين يكفي أن نبين أن مربعيهما متساويين

$$\left(\sqrt{a+\sqrt{a^2-b^2}}\right)^2 = a + \sqrt{a^2-b^2}$$

$$\left(\frac{\sqrt{2}}{2}(\sqrt{a-b}+\sqrt{a+b})\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 \times (\sqrt{a-b}+\sqrt{a+b})^2$$

$$= \frac{2}{4} \times (\sqrt{a-b}+\sqrt{a+b})^2 = \frac{1}{2} \times ((\sqrt{a-b})^2 + 2\sqrt{a-b}\sqrt{a+b} + (\sqrt{a+b})^2)$$

$$= \frac{2}{4} \times (\sqrt{a-b}+\sqrt{a+b})^2 = \frac{1}{2} \times (a-b+2\sqrt{(a-b)(a+b)}+a+b)$$

$$= \frac{1}{2} \times (2a+2\sqrt{(a-b)(a+b)}) = a + \sqrt{(a-b)(a+b)} = a + \sqrt{a^2-b^2}$$

مجموع عددين موجبين a و b اذن هو عدد موجب ومنه :

$$u = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$$

$v^2 = 36$ يعني $v = \sqrt{36}$ أو $v = -\sqrt{36}$ ولكن نعلم أن : $v = a - b$

ونلاحظ أن : $a > b$ اذن هو عدد موجب ومنه : $v = \sqrt{36} = 6$

(4) استنتاج كتابة للعددين a و b :

$$\begin{cases} a+b=2\sqrt{10} \\ a-b=6 \end{cases} \text{ يعني } \begin{cases} u=2\sqrt{10} \\ v=6 \end{cases} \text{ وجدنا : } \begin{cases} a+b=2\sqrt{10} \\ a-b=6 \end{cases} \text{ وجمع المعادلتين طرف}$$

$$a = \frac{6+2\sqrt{10}}{2} = 3+\sqrt{10} \text{ أي } 2a = 6+2\sqrt{10} \text{ لطرف نجد :}$$

$$\text{ولدينا : } a+b=2\sqrt{10} \text{ يعني } b=2\sqrt{10}-a \text{ يعني } b=2\sqrt{10}-3-\sqrt{10} \text{ يعني } b=\sqrt{10}-3$$

$$\text{تمرين 26: (1) بين أن : } \sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}} = \sqrt{10}$$

$$(2) \text{ بين أن : } \sqrt{\frac{6+\sqrt{31}}{2}} + \sqrt{\frac{6-\sqrt{31}}{2}} = \sqrt{6+\sqrt{5}}$$

$$(3) \text{ بين أن : } \sqrt{9-\sqrt{79}} + \sqrt{9+\sqrt{79}} = \sqrt{18+\sqrt{8}}$$

$$\text{الجواب: (1) } \sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}} = \sqrt{10} ?$$

$$\text{نضع : } B = \sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}}$$

$$\text{ونحسب } B^2 = (\sqrt{3+\sqrt{5}})^2 + 2\sqrt{3+\sqrt{5}}\sqrt{3-\sqrt{5}} + (\sqrt{3-\sqrt{5}})^2$$

$$B^2 = 3+\sqrt{5} + 2\sqrt{(3+\sqrt{5})(3-\sqrt{5})} + 3-\sqrt{5}$$

$$B^2 = 6+2\sqrt{3^2-(\sqrt{5})^2} = 6+2\sqrt{9-5} = 6+2\sqrt{4} = 6+4 = 10$$

$$B^2 = 10 \text{ يعني } B = \sqrt{10} \text{ أو } B = -\sqrt{10}$$

$$\text{ونعلم أن } B > 0 \text{ اذن } B = \sqrt{10}$$

$$(2) \text{ نضع : } B = \sqrt{\frac{6+\sqrt{31}}{2}} + \sqrt{\frac{6-\sqrt{31}}{2}}$$

$$\text{ونحسب } B^2 = \left(\sqrt{\frac{6+\sqrt{31}}{2}}\right)^2 + 2\sqrt{\frac{6+\sqrt{31}}{2}}\sqrt{\frac{6-\sqrt{31}}{2}} + \left(\sqrt{\frac{6-\sqrt{31}}{2}}\right)^2$$

$$B^2 = \frac{6+\sqrt{31}}{2} + 2\sqrt{\left(\frac{6+\sqrt{31}}{2}\right)\left(\frac{6-\sqrt{31}}{2}\right)} + \frac{6-\sqrt{31}}{2}$$

$$B^2 = 6+2\sqrt{\frac{36-1}{4}} = 6+2\sqrt{\frac{5}{4}} = 6+\sqrt{5}$$

$$B^2 = 6+\sqrt{5} \text{ يعني } B = \sqrt{6+\sqrt{5}} \text{ أو } B = -\sqrt{6+\sqrt{5}}$$

$$\text{ونعلم أن } B > 0 \text{ اذن } B = \sqrt{6+\sqrt{5}}$$

$$\text{ومنه } \sqrt{\frac{6+\sqrt{31}}{2}} + \sqrt{\frac{6-\sqrt{31}}{2}} = \sqrt{6+\sqrt{5}}$$

$$(3) \sqrt{9-\sqrt{79}} + \sqrt{9+\sqrt{79}} = \sqrt{18+\sqrt{8}} ?$$

$$\text{نضع : } B = \sqrt{9-\sqrt{79}} + \sqrt{9+\sqrt{79}}$$

$$\text{ونحسب } B^2 = (\sqrt{9-\sqrt{79}})^2 + 2\sqrt{9-\sqrt{79}}\sqrt{9+\sqrt{79}} + (\sqrt{9+\sqrt{79}})^2$$

$$B^2 = 9-\sqrt{79} + 2\sqrt{(9-\sqrt{79})(9+\sqrt{79})} + 9+\sqrt{79}$$

$$B^2 = 18+2\sqrt{81-79} = 18+\sqrt{8}$$

$$B^2 = 18+\sqrt{8} \text{ يعني } B = \sqrt{18+\sqrt{8}} \text{ أو } B = -\sqrt{18+\sqrt{8}}$$

$$\text{ونعلم أن } B > 0 \text{ اذن } B = \sqrt{18+\sqrt{8}}$$

$$\text{ومنه } \sqrt{9-\sqrt{79}} + \sqrt{9+\sqrt{79}} = \sqrt{18+\sqrt{8}}$$

« c'est en forgeant que l'on devient forgeron » dit un proverbe.
c'est en s'entraînant régulièrement aux calculs et exercices
que l'on devient un mathématicien



$$K = (x-2)(3x-4) + x^3 - 8$$

$$K = (x-2)(3x-4) + (x-2)(x^2+2x+4)$$

$$K = (x-2)(3x-4+x^2+2x+4) = (x-2)(x^2+5x)$$

$$K = x(x-2)(x+5)$$

$$R = x^2 - 6x + 8 = x^2 - 6x + 9 - 1 = x^2 - 2 \times 3x + 3^2 - 1$$

$$R = (x-3)^2 - 1^2 = (x-3-1)(x-3+1) = (x-4)(x-2)$$

$$\text{تمرين 23: } a \in \mathbb{R} \text{ نضع : } A = (a+1)^2 - (a-1)^2$$

$$(1) \text{ أحسب وبسط } A \text{ استنتج تبسيطا للعدد : } (9999999)^2 - (9999997)^2$$

$$\text{الجواب: (1) } A = (a+1)^2 - (a-1)^2 = a^2 + 2a + 1 - (a^2 - 2a + 1)$$

$$A = a^2 + 2a + 1 - a^2 + 2a - 1 = 4a$$

$$(2) \text{ لنأخذ : } a = 9999998$$

$$\text{اذن : } A = (9999999)^2 - (9999997)^2 = 4 \times 9999998$$

$$\text{ومنه : } (9999999)^2 - (9999997)^2 = 39999992$$

$$\text{تمرين 24: نضع : } B = \sqrt{6-2\sqrt{5}} - \sqrt{6+2\sqrt{5}}$$

$$(1) \text{ حدد اشارة العدد } B$$

$$(2) \text{ أحسب } B^2 \text{ استنتج كتابة مبسطة للعدد } B$$

$$\text{الجواب: (1) } B = \sqrt{6-2\sqrt{5}} - \sqrt{6+2\sqrt{5}}$$

$$\text{نلاحظ أن : } \sqrt{6-2\sqrt{5}} < \sqrt{6+2\sqrt{5}} \text{ ومنه } 6-2\sqrt{5} < 6+2\sqrt{5}$$

$$\text{وبالتالي : } B < 0$$

$$(2) B^2 = (\sqrt{6-2\sqrt{5}} - \sqrt{6+2\sqrt{5}})^2$$

$$B^2 = (\sqrt{6-2\sqrt{5}})^2 - 2\sqrt{6-2\sqrt{5}}\sqrt{6+2\sqrt{5}} + (\sqrt{6+2\sqrt{5}})^2$$

$$B^2 = 6-2\sqrt{5} - 2\sqrt{(6-2\sqrt{5})(6+2\sqrt{5})} + 6+2\sqrt{5}$$

$$B^2 = 12-2\sqrt{6^2-(2\sqrt{5})^2} = 12-2\sqrt{36-20} = 12-2\sqrt{16}$$

$$B^2 = 12-2 \times 4 = 4$$

$$B^2 = 4 \text{ يعني } B = \sqrt{4} \text{ أو } B = -\sqrt{4}$$

$$\text{يعني } B = 2 \text{ أو } B = -2 \text{ ونعلم أن } B < 0 \text{ اذن } B = -2$$

$$\text{تمرين 25: نضع : } a = \sqrt{19+6\sqrt{10}} \text{ و } b = \sqrt{19-6\sqrt{10}}$$

$$(1) \text{ بين أن : } a \times b = 1$$

$$(2) \text{ نضع : } u = a+b \text{ و } v = a-b$$

$$\text{أحسب } u^2 \text{ و } v^2$$

$$(3) \text{ استنتج كتابة للعددين } u \text{ و } v$$

$$(4) \text{ استنتج كتابة للعددين } a \text{ و } b$$

$$\text{الجواب: (1)}$$

$$ab = \sqrt{19+6\sqrt{10}}\sqrt{19-6\sqrt{10}} = \sqrt{(19+6\sqrt{10})(19-6\sqrt{10})}$$

$$ab = \sqrt{19^2 - (6\sqrt{10})^2} = \sqrt{361 - 360} = \sqrt{1} = 1$$

$$(2) u = a+b \text{ و } v = a-b$$

$$u^2 = (a+b)^2 = a^2 + b^2 + 2ab = a^2 + b^2 + 2 \times 1$$

$$u^2 = 19+6\sqrt{10} + 19-6\sqrt{10} + 2 \times 1 = 40$$

$$v^2 = (a-b)^2 = a^2 + b^2 - 2ab = a^2 + b^2 - 2 \times 1$$

$$v^2 = 19+6\sqrt{10} + 19-6\sqrt{10} - 2 \times 1 = 36$$

$$(3) \text{ استنتج كتابة للعددين } u \text{ و } v :$$

$$u^2 = 40 \text{ يعني } u = \sqrt{40} \text{ أو } u = -\sqrt{40}$$

$$\text{ولكن نعلم أن : } u = a+b$$