1) – لنثبت أن ABC مثلث قائم الزاوية.

لدينا : و $\left.\begin{array}{l} AB^2 = 6^2 = 36 \\ AC^2 = 8^2 = 64 \\ BC^2 = 10^2 = 100 \end{array}\right\}$ ، إذن : $BC^2 = AB^2 + AC^2$.

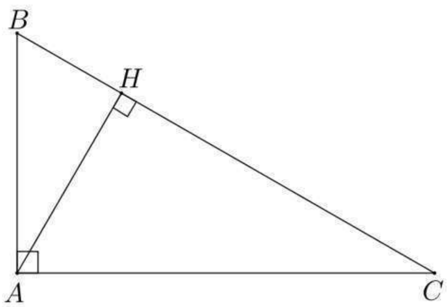
وحسب مبرهنة فيتاغورس المباشرة فإن ABC مثلث قائم الزاوية في A .

2) – لنحسب النسب المثلثية للزاوية $A\hat{B}C$:

*/ لدينا ABC مثلث قائم الزاوية في A .

إذن : و $\left.\begin{array}{l} \cos A\hat{B}C = \dfrac{AB}{BC} \\ \sin A\hat{B}C = \dfrac{AC}{BC} \\ \tan A\hat{B}C = \dfrac{AC}{AB} \end{array}\right\}$ ، أي : و $\left.\begin{array}{l} \cos A\hat{B}C = \dfrac{6}{10} \\ \sin A\hat{B}C = \dfrac{8}{10} \\ \tan A\hat{B}C = \dfrac{8}{6} \end{array}\right\}$ ، إذن : و $\left.\begin{array}{l} \boxed{\cos A\hat{B}C = \dfrac{3}{5}} \\ \boxed{\sin A\hat{B}C = \dfrac{4}{5}} \\ \boxed{\tan A\hat{B}C = \dfrac{4}{3}} \end{array}\right\}$.

3) – الشكل :



4) – */ لنحسب : AH .

بما أن H المسقط العمودي للنقطة A على المستقيم (BC) ، فإن : ABH مثلث قائم الزاوية في H .

و منه فإن : $\sin A\hat{B}H = \dfrac{AH}{AB}$ ، أي : $\sin A\hat{B}H = \dfrac{AH}{6}$

و بما أن : $A\hat{B}H = A\hat{B}C$ (نفس الزاوية) ، فإن : $\sin A\hat{B}H = \sin A\hat{B}C$

أي : $\dfrac{AH}{6} = \dfrac{4}{5}$ يعني أن : $AH = \dfrac{6\times 4}{5}$

و بالتالي فإن : $\boxed{AH = \dfrac{24}{5}\,cm}$.

*/ لنحسب : CH .

بما أن H المسقط العمودي للنقطة A على المستقيم (BC) ، فإن : ACH مثلث قائم الزاوية في H .

إذن حسب مبرهنة فيتاغورس المباشرة فإن : $AC^2 = AH^2 + CH^2$ ، أي : $8^2 = \left(\dfrac{24}{5}\right)^2 + CH^2$.

و منه فإن :

$$CH^2 = 8^2 - \left(\frac{24}{5}\right)^2 = 64 - \frac{576}{25} = \frac{1600-576}{25} = \frac{1024}{25}.$$

و بما أن : $CH > 0$ فإن : $CH = \sqrt{\dfrac{1024}{25}}$ ، و بالتالي فإن : $\boxed{CH = \dfrac{32}{5}\,cm}$.

1) – لنبسط ما يلي :

$$A = \cos\alpha(\sin\alpha + \cos\alpha) - \sin\alpha(\cos\alpha - \sin\alpha)$$
$$= \cos\alpha\times\sin\alpha + \cos^2\alpha - \sin\alpha\times\cos\alpha + \sin^2\alpha$$
$$= \cos\alpha\times\sin\alpha - \cos\alpha\times\sin\alpha + \cos^2\alpha + \sin^2\alpha$$
$$= 0 + 1$$
$$= 1$$

$$B = \frac{1}{1+\sin\alpha} + \frac{1}{1-\sin\alpha} - \frac{2}{\cos^2\alpha}$$
$$= \frac{(1-\sin\alpha)+(1+\sin\alpha)}{(1-\sin\alpha)(1+\sin\alpha)} - \frac{2}{\cos^2\alpha}$$
$$= \frac{1-\sin\alpha+1+\sin\alpha}{1^2-\sin^2\alpha} - \frac{2}{\cos^2\alpha}$$
$$= \frac{2}{1-\sin^2\alpha} - \frac{2}{\cos^2\alpha}$$
$$= \frac{2}{\cos^2\alpha} - \frac{2}{\cos^2\alpha}$$
$$= 0$$

$$C = \cos^4\alpha - \sin^4\alpha - \cos^2\alpha + 3\sin^2$$
$$= \left(\cos^2\alpha\right)^2 - \left(\sin^2\alpha\right)^2 - \cos^2\alpha + 3\sin^2$$
$$= \left(\cos^2\alpha + \sin^2\alpha\right)\left(\cos^2\alpha - \sin^2\right) - \cos^2\alpha + 3\sin^2$$
$$= 1\times\left(\cos^2\alpha - \sin^2\right) - \cos^2\alpha + 3\sin^2\alpha$$
$$= \cos^2\alpha - \sin^2 - \cos^2\alpha + 3\sin^2\alpha$$
$$= 2\sin^2\alpha$$

$$D = \sin\alpha\times\sqrt{1-\cos\alpha}\times\sqrt{1+\cos\alpha} + \cos^2\alpha$$
$$= \sin\alpha\times\sqrt{(1-\cos\alpha)(1+\cos\alpha)} + \cos^2\alpha$$
$$= \sin\alpha\times\sqrt{1^2-\cos^2\alpha} + \cos^2\alpha$$
$$= \sin\alpha\times\sqrt{1-\cos^2\alpha} + \cos^2\alpha$$
$$= \sin\alpha\times\sqrt{\sin^2\alpha} + \cos^2\alpha$$
$$= \sin\alpha\times\sin\alpha + \cos^2\alpha$$
$$= \sin^2\alpha + \cos^2\alpha$$
$$= 1$$

$$E=(\cos\alpha+\sin\alpha)^2+(\cos\alpha-\sin\alpha)^2$$
$$=\cos^2\alpha+2\cos\alpha\times\sin\alpha+\sin^2\alpha+\cos^2\alpha-2\cos\alpha\times\sin\alpha+\sin^2\alpha$$
$$=2\cos^2\alpha+2\sin^2\alpha$$
$$=2\left(\cos^2\alpha+\sin^2\alpha\right)$$
$$=2\times1$$
$$=2$$

$$F=\sqrt{2}\times\sin^2\alpha+2\sin 45^\circ\times\cos^2\alpha$$
$$=\sqrt{2}\times\sin^2\alpha+2\times\frac{\sqrt{2}}{2}\times\cos^2\alpha$$
$$=\sqrt{2}\times\sin^2\alpha+\sqrt{2}\times\cos^2\alpha$$
$$=\sqrt{2}\left(\sin^2\alpha+\cos^2\alpha\right)$$
$$=\sqrt{2}\times1$$
$$=\sqrt{2}$$

2)– */ لنبين أن : $\frac{\cos^4\alpha-\sin^4\alpha}{\cos^2\alpha-\sin^2\alpha}=1$.

لدينا :

$$\frac{\cos^4\alpha-\sin^4\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{\left(\cos^2\alpha\right)^2-\left(\sin^2\alpha\right)^2}{\cos^2\alpha-\sin^2\alpha}$$
$$=\frac{\left(\cos^2\alpha+\sin^2\alpha\right)\left(\cos^2\alpha-\sin^2\alpha\right)}{\cos^2\alpha-\sin^2\alpha}$$
$$=\frac{1\times\left(\cos^2\alpha-\sin^2\alpha\right)}{\cos^2\alpha-\sin^2\alpha}$$
$$=\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha}$$
$$=1$$

*/ لنبين أن : $\frac{1-\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1+\cos\alpha}$.

لدينا : و $\left.\begin{array}{r}(1-\cos\alpha)(1+\cos\alpha)=1-\cos^2\alpha=\sin^2\alpha\\ \sin\alpha\times\sin\alpha=\sin^2\alpha\end{array}\right\}$

إذن : $(1-\cos\alpha)(1+\cos\alpha)=\sin\alpha\times\sin\alpha$

و بالتالي فإن : $\frac{1-\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1+\cos\alpha}$

*/ لنبين أن : $\sqrt{1-\sin\alpha}\times\sqrt{1+\cos\alpha}=\cos\alpha$.

لدينا :

$$\sqrt{1-\sin\alpha}\times\sqrt{1+\sin\alpha}=\sqrt{(1-\sin\alpha)(1+\sin\alpha)}$$
$$=\sqrt{1^2-\sin^2\alpha}$$
$$=\sqrt{1-\sin^2\alpha}$$
$$=\sqrt{\cos^2\alpha}$$
$$=\cos\alpha$$

إذن : $\boxed{\sqrt{1-\sin\alpha}\times\sqrt{1+\cos\alpha}=\cos\alpha}$

*/ لنبين أن : $\sin^2\alpha=\frac{\tan^2}{1+\tan^2}$

لدينا :

$$\frac{\tan^2\alpha}{1+\tan^2\alpha}=\frac{\frac{\sin^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}}$$
$$=\frac{\sin^2\alpha}{\cos^2\alpha}\times\frac{\cos^2\alpha}{1}$$
$$=\sin^2\alpha$$

إذن : $\boxed{\sin^2\alpha=\frac{\tan^2}{1+\tan^2}}$.

*/ لنبين أن : $1+\tan^2\alpha=\frac{1}{\cos^2\alpha}$.

لدينا :

$$1+\tan^2\alpha=1+\frac{\sin^2\alpha}{\cos^2\alpha}$$
$$=\frac{\cos^2\alpha}{\cos^2\alpha}+\frac{\sin^2\alpha}{\cos^2\alpha}$$
$$=\frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha}$$
$$=\frac{1}{\cos^2\alpha}$$

إذن : $\boxed{1+\tan^2\alpha=\frac{1}{\cos^2\alpha}}$.

3) – أ) -- */ حساب $\cos\alpha$:

لدينا : $\sin^2\alpha+\cos^2\alpha=1$ يعني أن :

$$\cos^2\alpha=1-\sin^2\alpha$$

$$\cos^2\alpha=1-\left(\frac{\sqrt{3}}{2}\right)^2$$

$$\cos^2\alpha=1-\frac{3}{4}$$

$$\cos^2\alpha=\frac{4-3}{4}$$

$$\cos^2\alpha=\frac{1}{4}$$

و بما أن : $0<\cos\alpha<1$ فإن : $\cos\alpha=\sqrt{\frac{1}{4}}$

أي : $\boxed{\cos\alpha=\frac{1}{2}}$

*/ حساب $\tan\alpha$:

نعلم أن : $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}$

و منه فإن : $\tan\alpha=\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ و بالتالي : $\boxed{\tan\alpha=\sqrt{3}}$

ب) -- */ حساب :

$\sin\left(90^\circ-\alpha\right)$ و $\cos\left(90^\circ-\alpha\right)$ و $\tan\left(90^\circ-\alpha\right)$

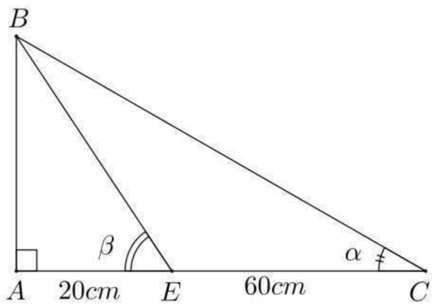
لدينا : $\left(90^\circ-\alpha\right)+\alpha=90^\circ+\alpha-\alpha=90^\circ$

إذن :

$$\sin\left(90^\circ-\alpha\right)=\cos\alpha=\frac{1}{2}$$

$$\cos\left(90^\circ-\alpha\right)=\sin\alpha=\frac{\sqrt{3}}{2}$$

$$\tan\left(90^\circ-\alpha\right)=\frac{1}{\tan\alpha}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$



لنحسب : AB .

*/ لدينا من خلال الشكل :

ABC مثلث قائم الزاوية في A و ABE مثلث قائم الزاوية في A .

إذن : $\left.\begin{array}{l}\tan A\hat{C}B=\dfrac{AB}{AC}\\ \tan A\hat{E}B=\dfrac{AB}{AE}\end{array}\right\}$ و

أي : $\left.\begin{array}{l}\tan\alpha=\dfrac{AB}{80}\\ \tan\beta=\dfrac{AB}{20}\end{array}\right\}$ و .

و بما أن : $\alpha+\beta=90^\circ$ فإن : $\tan\alpha=\dfrac{1}{\tan\beta}$

أي : $\dfrac{AB}{80}=\dfrac{1}{\frac{AB}{20}}$

و منه فإن : $\dfrac{AB}{80}=\dfrac{20}{AB}$ ، يعني أن : $AB^2=1600$

و بما أن : $AB>0$ فإن : $AB=\sqrt{1600}$ ، و بالتالي فإن : $\boxed{AB=40\ cm}$.

1) – */ حساب : $\cos A\hat{B}C$.

لدينا : $\cos^2 A\hat{B}C + \sin^2 A\hat{B}C = 1$ ، يعني أن $\cos^2 A\hat{B}C = 1 - \sin^2 A\hat{B}C$

أي : $\cos^2 A\hat{B}C = 1 - \left(\dfrac{3}{5}\right)^2 = 1 - \dfrac{9}{25} = \dfrac{25}{25} - \dfrac{9}{25} = \dfrac{16}{25}$

و بما أن : $0 < \cos A\hat{B}C < 1$ فإن : $\cos A\hat{B}C = \sqrt{\dfrac{16}{25}}$ ، و بالتالي فإن : $\boxed{\cos A\hat{B}C = \dfrac{4}{5}}$.

*/ حساب : $\tan A\hat{B}C$.

لدينا : $\tan A\hat{B}C = \dfrac{\sin A\hat{B}C}{\cos A\hat{B}C}$ ، أي : $\tan A\hat{B}C = \dfrac{\frac{3}{5}}{\frac{4}{5}} = \dfrac{3}{5} \times \dfrac{5}{4} = \dfrac{3}{4}$ ، إذن : $\boxed{\tan A\hat{B}C = \dfrac{3}{4}}$.

2) – حساب : AB و AC .

لدينا ABC مثلث قائم الزاوية في A .

إذن : و $\left.\begin{array}{l}\cos A\hat{B}C = \dfrac{AB}{BC} \\ \sin A\hat{B}C = \dfrac{AC}{BC}\end{array}\right\}$ ، أي : و $\left.\begin{array}{l}\dfrac{4}{5} = \dfrac{AB}{15} \\ \dfrac{3}{5} = \dfrac{AC}{BC}\end{array}\right\}$

يعني أن : و $\left.\begin{array}{l}AB = \dfrac{60}{5} \\ AC = \dfrac{45}{5}\end{array}\right\}$ ، و بالتالي فإن : و $\left.\begin{array}{l}\boxed{AB = 12\ cm} \\ \boxed{AC = 9\ cm}\end{array}\right\}$.

---

1) – لنحسب : $A = 2\cos 15^\circ + \cos^2 36^\circ - 2\sin 75^\circ + \cos^2 54^\circ$.

لدينا : و $\left.\begin{array}{l}15^\circ + 75^\circ = 90^\circ \\ 36^\circ + 54^\circ = 90^\circ\end{array}\right\}$ ، إذن : و $\left.\begin{array}{l}\cos 15^\circ = \sin 75^\circ \\ \cos 36^\circ = \cos 54^\circ\end{array}\right\}$

و منه فإن :

$$\begin{aligned}A &= 2\cos 15^\circ + \cos^2 36^\circ - 2\sin 75^\circ + \cos^2 54^\circ \\ &= 2\sin 75^\circ + \sin^2 54^\circ - 2\sin 75^\circ + \cos^2 54^\circ \\ &= 2\sin 75^\circ - 2\sin 75^\circ + \sin^2 54^\circ + \cos^2 54^\circ \\ &= 0 + 1 \\ &= 1\end{aligned}$$

2) – لنحسب : $B=\cos^2 28^\circ-\sin^2 51^\circ+\cos^2 62^\circ+\cos^2 39^\circ$.

لدينا : $\left.\begin{array}{l}28^\circ+62^\circ=90^\circ\\51^\circ+39^\circ=90^\circ\end{array}\right\}$ و ، إذن : $\left.\begin{array}{l}\cos 28^\circ=\sin 62^\circ\\\sin 51^\circ=\cos 39^\circ\end{array}\right\}$

و منه فإن :

$$\begin{aligned}B&=\cos^2 28^\circ-\sin^2 51^\circ+\cos^2 62^\circ+\cos^2 39^\circ\\&=\sin^2 62^\circ-\cos^2 39^\circ+\cos^2 62^\circ+\cos^2 39^\circ\\&=\sin 62^\circ+\cos^2 62^\circ-\cos^2 39^\circ+\cos^2 39^\circ\\&=\ 1\ +\ 0\\&=1\end{aligned}$$

3) – لنحسب : $C=\tan 73^\circ\times\tan 17^\circ-\sin^2 40^\circ-\sin^2 50^\circ$.

لدينا : $\left.\begin{array}{l}73^\circ+17^\circ=90^\circ\\40^\circ+50^\circ=90^\circ\end{array}\right\}$ و ، إذن : و $\left.\begin{array}{l}\tan 73^\circ=\dfrac{1}{\tan 17^\circ}\\\sin 40^\circ=\cos 50^\circ\end{array}\right\}$

و منه فإن :

$$\begin{aligned}C&=\tan 73^\circ\times\tan 17^\circ-\sin^2 40^\circ-\sin^2 50^\circ\\&=\frac{1}{\tan 17^\circ}\times\tan 17^\circ-\cos^2 50^\circ-\sin^2 50^\circ\\&=\frac{\tan 17^\circ}{\tan 17^\circ}\ -\left(\cos^2 50^\circ+\sin^2 50^\circ\right)\\&=1-1\\&=0\end{aligned}$$