

تحت إشراف : الأستاذ :

بوشعيب المأنسي

تصحيح الفرض 04

Groupe B

من أجل التلميذة صالحة
جبوج

$$V_{n+1} = \frac{3^{n+1} \times 2U_n}{2U_n - 1}$$

$$V_{n+1} = \frac{3^n \times 3 \times 2U_n}{2U_n - 1}$$

$$V_{n+1} = \frac{6 \times 3^n U_n}{2U_n - 1}$$

$$V_{n+1} = 6 V_n$$

ونعلم V_n متتالية هندسية أساسها 6

$$V_n = V_0 \cdot 6^n \Leftrightarrow V_n = -\frac{1}{3} \times 6^n$$

$$V_n = \frac{3^n U_n}{2U_n - 1} \Leftrightarrow V_n(2U_n - 1) = 3^n U_n$$

$$\Leftrightarrow U_n(2V_n - 3^n) = V_n$$

$$\Leftrightarrow U_n = \frac{V_n}{2V_n - 3^n}$$

$$\Leftrightarrow U_n = \frac{-\frac{6^n}{3}}{-\frac{2 \times 6^n}{3} - \frac{3^{n+1}}{3}}$$

$$\Leftrightarrow U_n = \frac{6^n}{2 \times 6^n + 3^{n+1}}$$

$$U_n = \frac{2^n \times 2^n}{2^n(2^{n+1} + 3)}$$

$$U_n = \frac{2^n}{2^{n+1} + 3}$$

$$\lim_{n \rightarrow +\infty} U_n = \lim_{n \rightarrow +\infty} \frac{2^n}{2^n(2 + \frac{3}{2^n})}$$

$$= \lim_{n \rightarrow +\infty} \frac{1}{2 + \frac{3}{2^n}}$$

$$= \frac{1}{2}$$

" $2 > 1$; $\frac{3}{2^n} \rightarrow 0$; $2^n \rightarrow +\infty$ " $\frac{3}{2^n} = 0$

التفريغ 01

$$U_{n+1} = \frac{2U_n}{2U_n + 1} \quad (1)$$

$$U_{n+1} = \frac{2U_n + 1 - 1}{2U_n + 1}$$

$$U_{n+1} = 1 - \frac{1}{2U_n + 1}$$

- نبين أن $0 < U_n < \frac{1}{2}$ $\forall n \in \mathbb{N}$

$$\text{من أجل } n=0 \quad 0 < U_0 = \frac{1}{5} < \frac{1}{2} \quad (v)$$

نفترض أن $0 < U_n < \frac{1}{2}$

ونبين أن $0 < U_{n+1} < \frac{1}{2}$

$$0 < U_n < \frac{1}{2} \Rightarrow 1 < 2U_n + 1 < 2$$

$$\Rightarrow -1 < \frac{-1}{2U_n + 1} < -\frac{1}{2}$$

$$\Rightarrow 0 < U_{n+1} < \frac{1}{2}$$

وعليه $\forall n \in \mathbb{N} \quad 0 < U_{n+1} < \frac{1}{2}$ (2)

$$U_{n+1} - U_n = \frac{2U_n - 2U_n^2 - U_n}{2U_n + 1}$$

$$= \frac{U_n - 2U_n^2}{2U_n + 1}$$

$$= \frac{U_n(1 - 2U_n)}{2U_n + 1}$$

$$* \quad 0 < 1 - 2U_n < 1$$

$$0 < 2U_n + 1 < 3$$

$$U_{n+1} - U_n > 0$$

وعليه U_n متزايدة.

$$V_{n+1} = \frac{3^{n+1} U_{n+1}}{2U_{n+1} - 1} \quad (3)$$

$$V_{n+1} = \frac{3^{n+1} \times \frac{2U_n}{2U_n + 1}}{2 \times \frac{2U_n}{2U_n + 1} - 1}$$

$$(\vec{BD}, \vec{AC}) = -\frac{\pi}{2} [2\pi]$$

$$\frac{|c-a|}{|d-b|} = 1$$

$$|c-a| = |d-b|$$

$$(\vec{CA}, \vec{AB}) = \frac{\pi}{2} [2\pi] \Rightarrow AC = BD$$

إذن ABCD مربع.

التمرين 03

$$z_1 = -\frac{3}{2} + \frac{\sqrt{3}}{2}i$$

$$|z_1| = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{3}$$

$$\cos \theta = \frac{-3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$$

$$\sin \theta = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$

$$\theta = \frac{5\pi}{6}$$

$$z_1 = \left[\sqrt{3}; \frac{5\pi}{6}\right]$$

$$z_2 = 2 - 2i$$

$$|z_2| = \sqrt{4+4}$$

$$= 2\sqrt{2}$$

$$\cos \theta = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin \theta = -\frac{2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\theta = -\frac{\pi}{4}$$

$$z_2 = \left[2\sqrt{2}; -\frac{\pi}{4}\right]$$

التمرين 02

$$\frac{b-a}{b-c} = \frac{5+i+1+i}{5+i-3-7i}$$

$$= \frac{6+2i}{2-6i}$$

$$= i$$

$$\left|\frac{b-a}{b-c}\right| = 1$$

$$|b-a| = |b-c| \Leftrightarrow AB = BC$$

$$(\vec{CB}, \vec{AB}) = \frac{\pi}{2} [2\pi]$$

إذن ABC مثلث متساوي الساقين

وقاسم الزاوية في B.

②

$$ABCD \text{ متوازي للأضلاع} \Leftrightarrow \vec{AB} = \vec{DC}$$

$$\Leftrightarrow \text{aff}(\vec{AB}) = \text{aff}(\vec{DC})$$

$$\Leftrightarrow b-a = c-d$$

$$\Leftrightarrow d = c-b+a$$

$$\Leftrightarrow d = 3+7i-5-i+1+i$$

$$d = 5i-3$$

③

$$\frac{c-a}{d-b} = \frac{3+7i+1+i}{5i-3-5-i}$$

$$= \frac{4i+4}{4i-8}$$

$$= \frac{2i+1}{i-2} = \frac{(1+2i)(-2-i)}{5}$$

$$= -\frac{5i}{5}$$

$$= -i$$

$$\lim_{x \rightarrow +\infty} x+1 - \ln(x+3) \quad \text{حساب :}$$

$$\lim_{n \rightarrow +\infty} n+1 - \ln(n+3) = \lim_{n \rightarrow +\infty} (n+3) \left(\frac{n+1}{n+3} - \frac{\ln(n+3)}{(n+3)} \right)$$

$$= +\infty$$

لأن : $\lim_{n \rightarrow +\infty} \frac{n+1}{n+3} = 1$ و $\lim_{n \rightarrow +\infty} \frac{\ln(n+3)}{n+3} = 0$ " "

$$\lim_{x \rightarrow -3^+} f(x) \quad \text{حساب :}$$

$$\lim_{n \rightarrow -3^+} n+1 - \ln(n+3) = +\infty$$

" $\lim_{n \rightarrow -3^+} \ln(n+3) = -\infty$ " و $\lim_{n \rightarrow -3^+} n+1 = -2$ "

$$\lim_{n \rightarrow +\infty} f(n) = +\infty$$

$$\lim_{n \rightarrow +\infty} \frac{f(n)}{n} = \lim_{n \rightarrow +\infty} 1 + \frac{1}{n} - \frac{\ln(n+3)}{n}$$

$$= \lim_{n \rightarrow +\infty} 1 + \frac{1}{n} - \frac{\ln(n(1+\frac{3}{n}))}{n}$$

$$= \lim_{n \rightarrow +\infty} 1 + \frac{1}{n} - \frac{\ln(n)}{n} - \frac{\ln(1+\frac{3}{n})}{n}$$

$$= 1$$

" $\lim_{n \rightarrow +\infty} \frac{\ln(n)}{n} = 0$ " لأن : $\lim_{n \rightarrow +\infty} \frac{\ln(1+\frac{3}{n})}{n} = 0$ " "

$$\lim_{n \rightarrow +\infty} f(n) - n = \lim_{n \rightarrow +\infty} 1 - \ln(n+3)$$

$$= -\infty$$

" $\lim_{n \rightarrow +\infty} \ln(n+3) = +\infty$ "

لأن f يقبل فرعاً شاملياً باتجاه
المستقيم $y=2$

$$Z_1 Z_2 = \left[2\sqrt{2} \times \sqrt{3}; \frac{5\pi}{6} - \frac{\pi}{4} \right]$$

$$= \left[2\sqrt{6}; \frac{7\pi}{12} \right]$$

تعدد الشكل الجبري Z_1, Z_2

$$Z_1 Z_2 = \left(-\frac{3}{2} + \frac{\sqrt{3}}{2} i \right) (2 - 2i)$$

$$= -3 + \sqrt{3} + i(6 + \sqrt{3})$$

$$\cos \frac{7\pi}{12} = \frac{\sqrt{3}-3}{2\sqrt{6}}$$

$$= \frac{\sqrt{6}(\sqrt{3}-3)}{12}$$

$$= \frac{3\sqrt{2}-3\sqrt{6}}{12}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$\sin \frac{7\pi}{12} = \frac{6+\sqrt{3}}{2\sqrt{6}}$$

$$= \frac{\sqrt{6}(6+\sqrt{3})}{12}$$

$$= \frac{6\sqrt{6}+3\sqrt{2}}{12}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

التمرين 04

الجزء 1

تعدد D_f

$$D_f = \{ x \in \mathbb{R} / x+3 > 0 \}$$

$$D_f = \{ x \in \mathbb{R} / x > -3 \}$$

$$D_f =]-3; +\infty[$$

2- نبين أن $\forall n \in \mathbb{N} - 2 \leq u_n \leq 2$

مذاحل $n=0$ $-2 \leq u_0 = 2 \leq 2$ (V)

نفترض أن $-2 \leq u_n \leq 2$

ونريد أن $-2 \leq u_{n+1} \leq 2$

لدينا $-2 \leq u_n \leq 2$ و f تزايدية

على المجال $[-2, 2] \subset [-2, +\infty[$

إذن $f(-2) \leq f(u_n) \leq f(2)$ $f(2) = 3 - \ln 5 < 2$

$-2 \leq -1 \leq u_{n+1} \leq 2$

$-2 \leq u_{n+1} \leq 2$

وهذه $\forall n \in \mathbb{N} - 2 \leq u_n \leq 2$

3- رتبة u_n :

لدينا $u_0 > u_1$

نفترض أن $u_n > u_{n+1}$

ولنريد أن $u_{n+1} > u_{n+2}$

لدينا $u_n > u_{n+1}$

و f تزايدية على المجال $[-2, 2]$

$f(u_n) > f(u_{n+1})$

$u_{n+1} > u_{n+2}$

إذن $\forall n \in \mathbb{N} - u_n > u_{n+1}$

وعليه فإن u_n تناقصية

لدينا u_n تناقصية ومضغوطة بـ -2

إذن متقاربة

$u_0 \in [-2, 2]$ و f متصلة

$f([-2, 2]) = [-2, 3 - \ln(5)] \subset [-2, 2]$

إذن نهاية u_n هي حل للمعادلة $f(x) = x$

$f(x) = x \Leftrightarrow 1 - \ln(x+3) = 0$

$\ln(x+3) = \ln e$

$x = e - 3$

لذا $u_n = e - 3$

3- أ- حساب المشتقة:

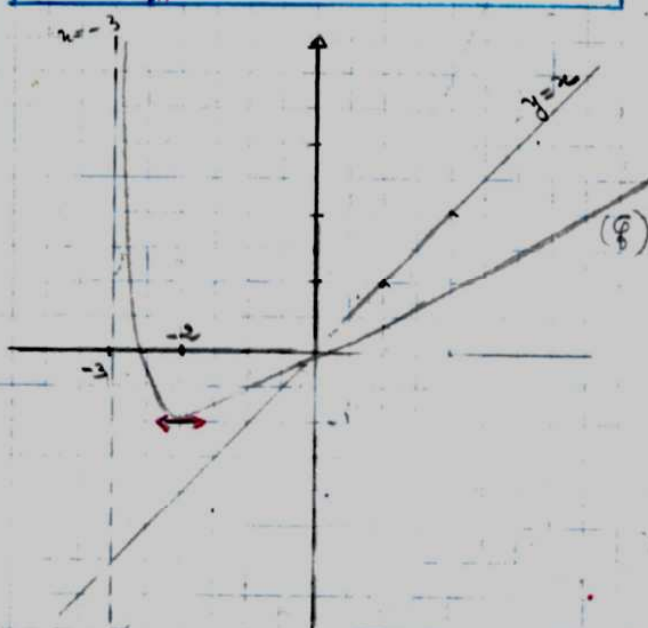
$f'(x) = 1 - \frac{(x+3)'}{x+3} + x'$

$f'(x) = -\frac{1}{x+3} + 1$

$f'(x) = \frac{x+3-1}{x+3}$

$f'(x) = \frac{x+2}{x+3}$

x	-3	-2	$+\infty$
$f'(x)$		-	+
$f(x)$	$+\infty$	-1	$+\infty$



1- الجزء 2:

$u_1 = f(u_0) \Leftrightarrow u_1 = 2 + 1 - \ln(5)$

$u_1 = 3 - \ln(5)$

$e < 5 \Leftrightarrow \ln e < \ln 5$ لدينا

$\Leftrightarrow -1 > -\ln 5$

$\Leftrightarrow 2 > 3 - \ln 5$

$\Leftrightarrow u_0 > u_1$