

تصحيح التمرين 1

1. الكتابة العقدية للإزاحة t

لتكن $M'(z')$ صورة النقطة $M(z)$ بالإزاحة t

$$t(M) = M' \quad \Leftrightarrow \overrightarrow{MM'} = \vec{w}$$

$$\Leftrightarrow z' - z = z_{\vec{w}}$$

$$\Leftrightarrow z' = z + z_{\vec{w}}$$

$$\Leftrightarrow z' = z + (2 - \sqrt{2}) + i(2 - \sqrt{6})$$

$$b = a + (2 - \sqrt{2}) + i(2 - \sqrt{6}) \quad .2$$

$$b = \sqrt{2} + i\sqrt{6} + 2 - \sqrt{2} + 2i - \sqrt{6}i$$

$$b = 2 + 2i$$

$$a = \sqrt{2} + i\sqrt{6} = 2\sqrt{2} \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = 2\sqrt{2} \left(\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right) \quad .3$$

$$b = 2 + 2i = 2\sqrt{2} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = 2\sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)$$

$$c = \frac{a}{b} = \frac{2\sqrt{2} \left(\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right)}{2\sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)} \quad \text{لدينا :}$$

$$c = \frac{2\sqrt{2}}{2\sqrt{2}} \left(\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \right) \quad \text{إن :}$$

$$c = \cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right) \quad \text{و منه :}$$

.4

$$\begin{aligned} c &= \frac{a}{b} \\ &= \frac{\sqrt{2} + i\sqrt{6}}{2 + 2i} \\ &= \frac{(\sqrt{2} + i\sqrt{6})(2 - 2i)}{(2 + 2i)(2 - 2i)} \\ &= \frac{2\sqrt{2} - 2i\sqrt{2} + 2i\sqrt{6} + 2\sqrt{6}}{8} \\ &= \left(\frac{\sqrt{6} + \sqrt{2}}{4} \right) + i \left(\frac{\sqrt{6} - \sqrt{2}}{4} \right) \end{aligned}$$

$$\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4} \text{ , } \cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} + \sqrt{2}}{4} \text{ : إذن } \begin{cases} c = \cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right) \\ c = \left(\frac{\sqrt{6} + \sqrt{2}}{4}\right) + i \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) \end{cases} \text{ لدينا : } 5$$

.6

$$\begin{aligned} c^{2007} &= \left(\cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right) \right)^{2007} \\ &= \cos\left(\frac{2007\pi}{12}\right) + i \sin\left(\frac{2007\pi}{12}\right) \\ &= \cos\left(\frac{-3\pi}{4}\right) + i \sin\left(\frac{-3\pi}{4}\right) \\ &= \cos\left(\frac{3\pi}{4}\right) - i \sin\left(\frac{3\pi}{4}\right) \\ &= \cos\left(\pi - \frac{\pi}{4}\right) - i \sin\left(\pi - \frac{\pi}{4}\right) \\ &= -\cos\left(\frac{\pi}{4}\right) - i \sin\left(\frac{\pi}{4}\right) \\ &= -\frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2} \end{aligned}$$

$$\left(\frac{2007\pi}{12} = \frac{-9\pi + 2016\pi}{12} = \frac{-9\pi}{12} + 168\pi = \frac{-3\pi}{4} + 2(84)\pi \right) \text{ (لاحظ أن :)}$$

تصحيح التمرين 2

1. (أ) لتكن $M'(z')$ صورة النقطة $M(z)$ بالتحاكي h الذي مركزه S ونسبته 3

$$\begin{aligned} h(M) = M' & \Leftrightarrow \overrightarrow{SM'} = 3\overrightarrow{SM} \\ \Leftrightarrow z' - s &= 3(z - s) \\ \Leftrightarrow z' &= 3z + 10 - 10i \end{aligned}$$

(ب) لدينا : $C(c)$ هي صورة النقطة $A(a)$ بالتحاكي h إذن :

$$\begin{aligned} c &= 3a + 10 - 10i \\ c &= 3(-2 + 4i) + 10 - 10i \\ c &= 4 + 2i \end{aligned}$$

و لدينا : $D(d)$ هي صورة النقطة $B(b)$ بالتحاكي h إذن :

$$\begin{aligned} d &= 3b + 10 - 10i \\ d &= 3(-4 + 2i) + 10 - 10i \\ d &= -2 - 4i \end{aligned}$$

(ج)

$$\begin{aligned} \frac{c-a}{b-a} \times \frac{b-d}{c-d} &= \frac{(4+2i)-(-2+4i)}{(-4+2i)-(-2+4i)} \times \frac{(-4+2i)-(-2-4i)}{(4+2i)-(-2-4i)} \\ &= \frac{6-2i}{-2-2i} \times \frac{-2+6i}{6+6i} \\ &= \frac{-12+36i+4i+12}{-12-12i-12i+12} \\ &= \frac{40i}{-24i} \\ &= \frac{-5}{3} \end{aligned}$$

بما أن $\frac{c-a}{b-a} \times \frac{b-d}{c-d} \in \mathbb{R}$ فإن النقط A و B و C و D متداورة

$$2. (أ) لدينا : p = \frac{a+c}{2} = \frac{-2+4i+4+2i}{2} = 1+3i$$

(ب)

✓

$$\begin{aligned}\frac{\omega - p}{b - d} &= \frac{(-2 + 2i) - (1 + 3i)}{(-4 + 2i) - (-2 - 4i)} \\ &= \frac{-3 - i}{-2 + 6i} \\ &= \frac{i(-1 + 3i)}{2(-1 + 3i)} \\ &= \frac{1}{2}i \\ &= \frac{1}{2}e^{i\frac{\pi}{2}}\end{aligned}$$

$$DB = 2P\Omega \text{ و منه } \frac{P\Omega}{DB} = \frac{1}{2} \text{ إذن } \left| \frac{\omega - p}{b - d} \right| = \frac{1}{2} \quad \checkmark \text{ لدينا :}$$

$$\arg\left(\frac{\omega - p}{b - d}\right) \equiv \frac{\pi}{2} [2\pi] \text{ إذن : } \arg\left(\frac{\overrightarrow{DB}, \overrightarrow{P\Omega}}{DB}\right) \equiv \frac{\pi}{2} [2\pi] \quad \checkmark \text{ ولدينا :}$$

تصحيح التمرين 3

$$1. \text{ لنحل في } \mathbb{C} \text{ المعادلة } z^2 - 4z + 8 = 0$$

$$\Delta = (-4)^2 - 4(1)(8) = -16 \text{ لدينا :}$$

بما أن $\Delta < 0$ فإن المعادلة تقبل حلين عقديين مترافقين :

$$z = \frac{-(-4) + i\sqrt{16}}{2(1)} \text{ أو } z = \frac{-(-4) - i\sqrt{16}}{2(1)}$$

$$\text{إذن : } z = 2 + 2i \text{ أو } z = 2 - 2i$$

$$\text{و منه : } S = \{2 - 2i, 2 + 2i\}$$

2. أ.

$$|z_A| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2} \quad \checkmark$$

$$\arg(z_A) \equiv \frac{\pi}{4} [2\pi] \text{ و منه : } z_A = 2\sqrt{2} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = 2\sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)$$

$$|z_B| = |\overline{z_A}| = |z_A| = 2\sqrt{2} \quad \checkmark$$

$$\begin{aligned}\arg(z_B) &\equiv \arg(\overline{z_A})[2\pi] \\ &\equiv -\arg(z_A)[2\pi] \\ &\equiv -\frac{\pi}{4}[2\pi]\end{aligned}$$

$$\frac{z_A}{z_B} = \frac{2\sqrt{2}e^{i\frac{\pi}{4}}}{2\sqrt{2}e^{-i\frac{\pi}{4}}} = 1.e^{i\left(\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right)} = 1.e^{i\frac{\pi}{2}} \quad \text{ب. لدينا :}$$

$$\frac{z_A - z_O}{z_B - z_O} = 1.e^{i\frac{\pi}{2}} \quad \text{إذن :}$$

$$OA = OB \quad \text{منه} \quad \frac{OA}{OB} = 1 \quad \text{إذن :} \quad \left| \frac{z_A - z_O}{z_B - z_O} \right| = 1 \quad \text{لدينا :} \quad \checkmark$$

$$\left(\overrightarrow{OB}, \overrightarrow{OA} \right) \equiv \frac{\pi}{2}[2\pi] \quad \text{إذن :} \quad \arg\left(\frac{z_A - z_O}{z_B - z_O} \right) \equiv \frac{\pi}{2}[2\pi] \quad \text{لدينا :} \quad \checkmark$$

و بالتالي المثلث OAB متساوي الساقين وقائم الزاوية في O

$$z_B - z_O = 2 - 2i \quad \text{و} \quad z_C - z_A = 2 - 2i \quad \text{ج. لدينا :}$$

$$\text{إذن} \quad z_C - z_A = z_B - z_O \quad \text{و منه} \quad \overrightarrow{AC} = \overrightarrow{OB} \quad \text{و بالتالي الرباعي} \quad OBCA \quad \text{متوازي أضلاع}$$

و بما أن $\overrightarrow{OA} \perp \overrightarrow{OB}$ فإن $OBCA$ مستطيل
و بما أن $OA = OB$ فإن $OBCA$ مربع.

$$z_D = iz_A = i(2 + 2i) = -2 + 2i \quad \text{و} \quad z_E = \frac{z_O + z_A}{2} = \frac{0 + 2 + 2i}{2} = 1 + i \quad \text{د. لدينا :}$$

$$\frac{z_C + z_D}{2} = \frac{4 - 2 + 2i}{2} = 1 + i$$

$$\text{بما أن :} \quad \frac{z_C + z_D}{2} = z_E \quad \text{فإن} \quad E \quad \text{منتصف} \quad [CD]$$

تصحيح التمرين 4

1. أ) لدينا :

$$\begin{aligned}\frac{c-b}{a-b} &= \frac{2i\sqrt{3}-3-i\sqrt{3}}{2-3-i\sqrt{3}} \\ &= \frac{-3+i\sqrt{3}}{-1-i\sqrt{3}} \\ &= \frac{-i\sqrt{3}(-1-i\sqrt{3})}{-1-i\sqrt{3}} \\ &= -i\sqrt{3} = \sqrt{3}e^{i\left(\frac{-\pi}{2}\right)}\end{aligned}$$

$$\left(\overrightarrow{BA}, \overrightarrow{BC}\right) \equiv \arg\left(\frac{c-b}{a-b}\right)[2\pi]$$

إذن :

$$\equiv \frac{-\pi}{2}[2\pi]$$

ب) بما أن المثلث ABC قائم الزاوية في B فإن $[AC]$ يمثل قطر الدائرة المحاطة بالمثلث ABC

$$\omega = \frac{a+c}{2} = \frac{2+2i\sqrt{3}}{2} = 1+i\sqrt{3} \text{ أي : } [AC] \text{ هو منتصف القطعة } [AC] \text{ إذن } \Omega \text{ مركز هذه الدائرة هو منتصف القطعة } [AC]$$

2. أ)

$$z_1 = \frac{1+i\sqrt{3}}{2}z_0 + 2 = 2 \quad \checkmark$$

$$z_2 = \frac{1+i\sqrt{3}}{2}z_1 + 2 = \frac{1+i\sqrt{3}}{2}a + 2 = 3+i\sqrt{3} = b \quad \checkmark$$

$$z_3 = \frac{1+i\sqrt{3}}{2}z_2 + 2 = \frac{1+i\sqrt{3}}{2}b + 2 = \frac{1+i\sqrt{3}}{2}(3+i\sqrt{3}) + 2 = 2+2i\sqrt{3} \quad \checkmark$$

$$z_4 = \frac{1+i\sqrt{3}}{2}z_3 + 2 = \frac{1+i\sqrt{3}}{2}(2+2i\sqrt{3}) + 2 = 2i\sqrt{3} = c \quad \checkmark$$

$$A_3A_4 = |z_4 - z_3| = 2 \quad A_2A_3 = |z_3 - z_2| = 2 \quad A_1A_2 = |z_2 - z_1| = 2 \quad \text{ب)}$$

$$A_1A_2 = A_2A_3 = A_3A_4 \text{ إذن :}$$

ج) ليكن $n \in \mathbb{N}$:

$$z_{n+1} - \omega = \frac{1+i\sqrt{3}}{2}z_n + 2 - 1 - i\sqrt{3} = \frac{1+i\sqrt{3}}{2}z_n + 1 - i\sqrt{3} = \frac{1+i\sqrt{3}}{2}(z_n - (1+i\sqrt{3})) = \frac{1+i\sqrt{3}}{2}(z_n - \omega)$$

(د) بما أن $z_{n+1} - \omega = e^{i\frac{\pi}{3}}(z_n - \omega)$ فإن A_{n+1} هي صورة A_n بالدوران R الذي مركزه Ω و زاويته $\frac{\pi}{3}$ (٥)

✓ نضع $v_n = z_n - \omega$

لدينا (v_n) هندسية أساسها $e^{i\frac{\pi}{3}}$ و حدها الأول $v_0 = -\omega$ إذن : $v_n = -\omega \left(e^{i\frac{\pi}{3}} \right)^n = -\omega e^{\frac{in\pi}{3}}$

و منه $z_n = \omega - \omega e^{\frac{in\pi}{3}}$

إذن : $z_{n+6} = \omega - \omega e^{\frac{i(n+6)\pi}{3}} = \omega - \omega e^{\frac{in\pi}{3}} e^{i2\pi} = \omega - \omega e^{\frac{in\pi}{3}} = z_n$

✓ $z_{2012} = z_{2+6(335)} = z_2 = 3 + i\sqrt{3}$

(و) نضع $d_n = A_n A_{n+1} = |z_{n+1} - z_n|$

بحساب d_{n+1} نجد $d_{n+1} = d_n$ إذن (d_n) ثابتة و منه $d_n = d_1 = A_1 A_2 = 2$ أي : $A_n A_{n+1} = 2$ (يمكنك كذلك استعمال خاصيات الدوران بطريقة أخرى)

تصحيح التمرين 5

$$|U| = \sqrt{(2 + \sqrt{3})^2 + 1^2} = \sqrt{8 + 4\sqrt{3}} = 2\sqrt{2 + \sqrt{3}} \quad (1) \quad \text{I.}$$

$$U = 2 + \sqrt{3} + i = 2 \left(1 + \frac{\sqrt{3}}{2} \right) + i \cdot 2 \cdot \frac{1}{2} = 2 \left(1 + \cos\left(\frac{\pi}{6}\right) \right) + i \cdot 2 \sin\left(\frac{\pi}{6}\right) \quad (2)$$

(3) I

$$\begin{aligned} \cos^2(\theta) &= \left(\frac{e^{i\theta} + e^{-i\theta}}{2} \right)^2 \\ &= \frac{e^{2i\theta} + 2e^{i\theta}e^{-i\theta} + e^{-2i\theta}}{4} \\ &= \frac{1}{2} \left(\frac{e^{2i\theta} + 1 + e^{-2i\theta}}{2} \right) \\ &= \frac{1}{2} \left(\frac{e^{2i\theta} + e^{-2i\theta}}{2} + 1 \right) = \frac{1}{2} (\cos(2\theta) + 1) \end{aligned}$$

و منه : $1 + \cos(2\theta) = 2\cos^2(\theta)$

(ب)

$$\begin{aligned}
 U &= 2 \left(1 + \cos \left(\frac{\pi}{6} \right) \right) + i \cdot 2 \sin \left(\frac{\pi}{6} \right) \\
 &= 2 \left(1 + \cos \left(2 \cdot \frac{\pi}{12} \right) \right) + i \cdot 2 \sin \left(2 \cdot \frac{\pi}{12} \right) \\
 &= 2 \times 2 \cos^2 \left(\frac{\pi}{12} \right) + i \cdot 2 \times 2 \cos \left(\frac{\pi}{12} \right) \sin \left(\frac{\pi}{12} \right) \\
 &= 4 \cos^2 \left(\frac{\pi}{12} \right) + i \cdot 4 \cos \left(\frac{\pi}{12} \right) \sin \left(\frac{\pi}{12} \right) \\
 &\text{بما أن } \cos \left(\frac{\pi}{12} \right) > 0 \text{ فإن الشكل المثلثي للعدد } U \text{ هو :}
 \end{aligned}$$

$$U = 4 \cos \left(\frac{\pi}{12} \right) \cdot \left(\cos \left(\frac{\pi}{12} \right) + i \sin \left(\frac{\pi}{12} \right) \right)$$

(ج)

$$\begin{aligned}
 U^6 &= \left(|U| \cdot \left(\cos \left(\frac{\pi}{12} \right) + i \sin \left(\frac{\pi}{12} \right) \right) \right)^6 \\
 &= |U|^6 \cdot \left(\cos \left(\frac{6\pi}{12} \right) + i \sin \left(\frac{6\pi}{12} \right) \right) \\
 &= \left(2\sqrt{2+\sqrt{3}} \right)^6 \cdot \left(\cos \left(\frac{\pi}{2} \right) + i \sin \left(\frac{\pi}{2} \right) \right) \\
 &= \left(2\sqrt{2+\sqrt{3}} \right)^6 \cdot i
 \end{aligned}$$

II. (1)

$$\begin{aligned}
 d - \omega &= 2(p - \omega) \\
 d &= 2p - 2\omega + \omega \\
 d &= 2p - \omega \\
 d &= 2(2 + \sqrt{3} + i) - \sqrt{3} \\
 d &= (4 + \sqrt{3}) + 2i
 \end{aligned}$$

$$|z - d| = 2\sqrt{2+\sqrt{3}} \quad \text{تكافئ} \quad |z - 4 - \sqrt{3} - 2i| = |U| \quad (1) \text{ لدينا :}$$

إن مجموعة النقط M هي الدائرة التي مركزها D و شعاعها $2\sqrt{2+\sqrt{3}}$

تصحيح التمرين 6

1. (أ)

$$z' - z_0 = e^{i\frac{2\pi}{3}} \cdot (z - z_0) \quad \checkmark$$

$$z' - 0 = \left(\frac{-1}{2} + i \frac{\sqrt{3}}{2} \right) \cdot (z - 0)$$

$$z' = \left(\frac{-1}{2} + i \frac{\sqrt{3}}{2} \right) \cdot z$$

✓ لدينا : $C(c)$ صورة $B(b)$ بالدوران r :

$$c = e^{i\frac{2\pi}{3}} \times b \quad \text{إن :}$$

$$\text{إن : } c = e^{i\frac{2\pi}{3}} \times e^{i\frac{-5\pi}{6}} \quad \text{و منه : } c = e^{-i\frac{\pi}{6}}$$

(ب)

✓

$$b = e^{-i\frac{5\pi}{6}}$$

$$= \cos\left(\frac{-5\pi}{6}\right) + i \sin\left(\frac{-5\pi}{6}\right)$$

$$= \cos\left(\frac{5\pi}{6}\right) - i \sin\left(\frac{5\pi}{6}\right)$$

$$= \cos\left(\pi - \frac{\pi}{6}\right) - i \sin\left(\pi - \frac{\pi}{6}\right)$$

$$= -\cos\left(\frac{\pi}{6}\right) - i \sin\left(\frac{\pi}{6}\right)$$

$$= \frac{-\sqrt{3}}{2} - i \frac{1}{2}$$

✓

$$\begin{aligned} c &= e^{-i\frac{\pi}{6}} \\ &= \cos\left(\frac{-\pi}{6}\right) + i \sin\left(\frac{-\pi}{6}\right) \\ &= \cos\left(\frac{\pi}{6}\right) - i \sin\left(\frac{\pi}{6}\right) \\ &= \frac{\sqrt{3}}{2} - i \frac{1}{2} \end{aligned}$$

$$d = \frac{(2)a + (-1)b + (2)c}{(2) + (-1) + (2)} = \frac{\sqrt{3}}{2} + i \frac{1}{2} \quad (أ) \quad 2.$$

(ب) (بعد الحساب) نجد $\frac{c-a}{b-a} \times \frac{b-d}{c-d} = 2 \in \mathbb{R}$ إذن النقط A و B و C و D متداورة

3.

✓

$$\begin{aligned} z' - a &= 2(z - a) \\ z' - i &= 2(z - i) \\ z' &= 2z - i \end{aligned}$$

✓

$$\begin{aligned} e &= 2d - i \\ e &= 2\left(\frac{\sqrt{3}}{2} + i \frac{1}{2}\right) - i \\ e &= \sqrt{3} \end{aligned}$$

$$\frac{d-c}{e-c} = \frac{1}{2} + i \frac{\sqrt{3}}{2} = 1 e^{i\frac{\pi}{3}} \quad (أ) \quad 4.$$

(ب)

✓ بما أن : $\left| \frac{d-c}{e-c} \right| = 1$ فإن : $CD = CE$

✓ و بما أن $\arg\left(\frac{d-c}{e-c}\right) \equiv \frac{\pi}{3} [2\pi]$ فإن : $\left(\overrightarrow{CE}, \overrightarrow{CD}\right) \equiv \frac{\pi}{3} [2\pi]$

و بالتالي المثلث CDE متساوي الأضلاع