

تصحيح التمرين 1:

(1)

$$\begin{aligned} I &= \int_0^1 (x^2 - x + 1) dx \\ &= \left[\frac{x^3}{3} - \frac{x^2}{2} + x \right]_0^1 \\ &= \left(\frac{1^3}{3} - \frac{1^2}{2} + 1 \right) - \left(\frac{0^3}{3} - \frac{0^2}{2} + 0 \right) \\ &= \frac{1}{3} - \frac{1}{2} + 1 \\ &= \frac{5}{6} \end{aligned}$$

(2)

$$\begin{aligned} J &= \int_0^{\frac{\pi}{2}} \sin(2x) dx \\ &= \left[-\frac{1}{2} \cos(2x) \right]_0^{\frac{\pi}{2}} \\ &= \left(-\frac{1}{2} \cos(\pi) \right) - \left(-\frac{1}{2} \cos(0) \right) \\ &= \left(-\frac{1}{2}(-1) \right) - \left(-\frac{1}{2}(1) \right) \\ &= 1 \end{aligned}$$

(3)

$$\begin{aligned} K &= \int_0^{\ln(2)} e^x dx \\ &= \left[e^x \right]_0^{\ln(2)} \\ &= \left(e^{\ln(2)} \right) - \left(e^0 \right) \\ &= (2) - (1) \\ &= 1 \end{aligned}$$

(4)

$$\begin{aligned} L &= \int_1^2 \frac{1}{x+1} dx \\ &= \int_1^2 \frac{(x+1)'}{x+1} dx \\ &= [\ln|x+1|]_1^2 \\ &= (\ln(3)) - (\ln(2)) \\ &= \ln\left(\frac{3}{2}\right) \end{aligned}$$

(5)

$$\begin{aligned} M &= \int_0^4 \sqrt{x+3} dx \\ &= \int_0^4 (x+3)^{\frac{1}{2}} dx \\ &= \int_0^4 (x+3)' (x+3)^{\frac{1}{2}} dx \\ &= \left[\frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\ &= \left[\frac{2}{3} (x+3)^{\frac{3}{2}} \right]_0^4 \\ &= \left(\frac{2}{3} (7)^{\frac{3}{2}} \right) - \left(\frac{2}{3} (3)^{\frac{3}{2}} \right) \\ &= \frac{2}{3} (7\sqrt{7} - 3\sqrt{3}) \end{aligned}$$

(6)

$$\begin{aligned}
 N &= \int_0^1 x (x^2 - 1)^4 dx \\
 &= \frac{1}{2} \int_0^1 (2x) (x^2 - 1)^4 dx \\
 &= \frac{1}{2} \int_0^1 (x^2 - 1)' (x^2 - 1)^4 dx \\
 &= \frac{1}{2} \left[\frac{(x^2 - 1)^5}{5} \right]_0^1 \\
 &= \frac{1}{2} \left(\frac{0^5}{5} - \frac{(-1)^5}{5} \right) \\
 &= \frac{1}{2} \times \frac{1}{5} \\
 &= \frac{1}{10}
 \end{aligned}$$

تصحيح التمرين 2:

x	$-\infty$	1	$+\infty$
$x-1$	$-$	0	$+$

(1)

$$\begin{aligned}
 I &= \int_0^2 |x - 1| dx \\
 &= \int_0^1 |x - 1| dx + \int_1^2 |x - 1| dx \\
 &= \int_0^1 (-x + 1) dx + \int_1^2 (x - 1) dx \\
 &= \left[-\frac{x^2}{2} + x \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^2 \\
 &= \left(\left(\frac{1}{2} \right) - (0) \right) + \left((0) - \left(\frac{-1}{2} \right) \right) \\
 &= 1
 \end{aligned}$$

(2)

x	0	$1 + \infty$
$\ln(x)$	-	0 +

$$\begin{aligned}
 J &= \int_{\frac{1}{e}}^e \frac{|\ln x|}{x} dx \\
 &= \int_{\frac{1}{e}}^1 \frac{|\ln x|}{x} dx + \int_1^e \frac{|\ln x|}{x} dx \\
 &= \int_{\frac{1}{e}}^1 \frac{-\ln x}{x} dx + \int_1^e \frac{\ln x}{x} dx \\
 &= -\int_{\frac{1}{e}}^1 \frac{1}{x} \ln x dx + \int_1^e \frac{1}{x} \ln x dx \\
 &= -\int_{\frac{1}{e}}^1 \ln'(x) \ln x dx + \int_1^e \ln'(x) \ln x dx \\
 &= -\left[\frac{\ln^2(x)}{2} \right]_{\frac{1}{e}}^1 + \left[\frac{\ln^2(x)}{2} \right]_1^e + \\
 &= -\left((0) - \frac{(-1)^2}{2} \right) + \left(\frac{\ln^2(2)}{2} - (0) \right) \\
 &= \frac{1 + \ln^2(2)}{2}
 \end{aligned}$$

تصحیح التمرین 3:

(1)

$$\begin{aligned}
 \mu &= \frac{1}{\frac{\pi}{4} - 0} \int_0^{\frac{\pi}{4}} \cos(2x) dx \\
 &= \frac{4}{\pi} \left[\frac{1}{2} \sin(2x) \right]_0^{\frac{\pi}{4}} \\
 &= \frac{4}{\pi} \left(\frac{1}{2} \sin\left(\frac{\pi}{2}\right) - \frac{1}{2} \sin(0) \right) \\
 &= \frac{4}{\pi} \left(\frac{1}{2} - 0 \right) \\
 &= \frac{2}{\pi}
 \end{aligned}$$

(2) أ. بما أن $x \mapsto \frac{x^4}{1+x^2}$ متصلة و موجبة على $[1, 2]$ (و $1 < 2$) فإن $\int_1^2 \frac{x^4}{1+x^2} dx \geq 0$

ب. بما أن $x \mapsto \ln(x)$ متصلة و سالبة على $\left[\frac{1}{e}, 1\right]$ (و $\frac{1}{e} < 1$) فإن $\int_{\frac{1}{e}}^1 \ln(x) dx \leq 0$

تصحيح التمرين 4 :

(1)

$$\begin{aligned} I &= \int_0^1 x (x^2 + 3)^2 dx \\ &= \frac{1}{2} \int_0^1 (2x) (x^2 + 3)^2 dx \\ &= \frac{1}{2} \int_0^1 (x^2 + 3)' (x^2 + 3)^2 dx \\ &= \frac{1}{2} \left[\frac{(x^2 + 3)^3}{3} \right]_0^1 \\ &= \frac{1}{2} \left(\frac{64}{3} - \frac{27}{3} \right) \\ &= \frac{37}{6} \end{aligned}$$

(2)

$$\begin{aligned} J &= \int_0^1 \frac{x-1}{(x^2-2x+3)^2} dx \\ &= \frac{1}{2} \int_0^1 \frac{2x-2}{(x^2-2x+3)^2} dx \\ &= \frac{1}{2} \int_0^1 \frac{(x^2-2x+3)'}{(x^2-2x+3)^2} dx \\ &= \frac{1}{2} \left[\frac{-1}{x^2-2x+3} \right]_0^1 \\ &= \frac{1}{2} \left(\left(\frac{-1}{2} \right) - \left(\frac{-1}{3} \right) \right) \\ &= \frac{-1}{12} \end{aligned}$$

(3)

$$\begin{aligned} K &= \int_0^2 \frac{2}{x+1} dx \\ &= 2 \int_0^2 \frac{1}{x+1} dx \\ &= 2 \int_0^2 \frac{(x+1)'}{x+1} dx \\ &= 2 [\ln|x+1|]_0^2 \\ &= 2((\ln(3)) - (\ln(1))) \\ &= 2\ln(3) \end{aligned}$$

(4)

$$\begin{aligned} L &= \int_1^2 \frac{x}{x+1} dx \\ &= \int_1^2 \frac{x+1-1}{x+1} dx \\ &= \int_1^2 \left(1 - \frac{1}{x+1}\right) dx \\ &= \int_1^2 \left(1 - \frac{(x+1)'}{x+1}\right) dx \\ &= [x - \ln|x+1|]_1^2 \\ &= (2 - \ln(3)) - (1 - \ln(2)) \\ &= 1 + \ln(2) - \ln(3) \\ &= 1 + \ln\left(\frac{2}{3}\right) \end{aligned}$$

(5)

$$\begin{aligned} M &= \int_1^e \frac{\ln^2(x)}{x} dx \\ &= \int_1^e \frac{1}{x} \ln^2(x) dx \\ &= \int_1^e \ln'(x) \ln^2(x) dx \\ &= \left[\frac{\ln^3(x)}{3} \right]_1^e \\ &= \frac{\ln^3(e)}{3} - \frac{\ln^3(1)}{3} \\ &= \frac{1}{3} \end{aligned}$$

تصحيح التمرين 5 :

(1) لنحسب $\int_1^e x \ln x dx$.

$$\begin{cases} U'(x) = x \\ V(x) = \ln(x) \end{cases} \quad \nearrow \quad \begin{cases} U(x) = \frac{x^2}{2} \\ V'(x) = \frac{1}{x} \end{cases} \quad \Downarrow$$

$$\begin{aligned} \int_1^e x \ln x dx &= \int_1^e U'(x) V(x) dx \\ &= [U(x) V(x)]_1^e - \int_1^e U(x) V'(x) dx \\ &= \left[\frac{x^2}{2} \ln(x) \right]_1^e - \int_1^e \frac{x^2}{2} \times \frac{1}{x} dx \\ &= \left[\frac{x^2}{2} \ln(x) \right]_1^e - \frac{1}{2} \int_1^e x dx \\ &= \left[\frac{x^2}{2} \ln(x) \right]_1^e - \frac{1}{2} \left[\frac{x^2}{2} \right]_1^e \\ &= \left(\frac{e^2}{2} \ln(e) - \frac{1^2}{2} \ln(1) \right) - \frac{1}{2} \left(\frac{e^2}{2} - \frac{1^2}{2} \right) \\ &= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} \\ &= \frac{e^2 + 1}{4} \end{aligned}$$

(2) لنحسب $\int_0^{\frac{\pi}{2}} x \cos x dx$.

$$\begin{cases} U'(x) = \cos x \\ V(x) = x \end{cases} \quad \nearrow \quad \begin{cases} U(x) = \sin x \\ V'(x) = 1 \end{cases} \quad \Downarrow$$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} x \cos x dx &= [x \sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x dx \\ &= [x \sin x]_0^{\frac{\pi}{2}} - [-\cos x]_0^{\frac{\pi}{2}} \\ &= \left(\left(\frac{\pi}{2} \right) - (0) \right) - ((0) - (-1)) \\ &= \frac{\pi}{2} - 1 \end{aligned}$$

(3) لنحسب $\int_1^e \ln x dx$.

لدينا : $\int_1^e \ln x dx = \int_1^e 1 \times \ln(x) dx$

$$\begin{cases} U'(x) = 1 \\ V(x) = \ln x \end{cases} \quad \nearrow \quad \begin{cases} U(x) = x \\ V'(x) = \frac{1}{x} \end{cases} \quad \Downarrow$$

$$\begin{aligned} \int_1^e \ln x dx &= [x \ln x]_1^e - \int_1^e x \times \frac{1}{x} dx \\ &= [x \ln x]_1^e - \int_1^e 1 dx \\ &= [x \ln x]_1^e - [x]_1^e \\ &= (e - 0) - (e - 1) \\ &= 1 \end{aligned}$$

(4) لنحسب $\int_0^1 \ln(x+1) dx$.

لدينا : $\int_0^1 \ln(x+1) dx = \int_0^1 1 \times \ln(x+1) dx$

$$\begin{cases} U'(x) = 1 \\ V(x) = \ln(x+1) \end{cases} \quad \nearrow \quad \begin{cases} U(x) = x+1 \\ V'(x) = \frac{(x+1)'}{x+1} = \frac{1}{x+1} \end{cases} \quad \Downarrow$$

$$\begin{aligned} \int_0^1 \ln(x+1) dx &= [(x+1) \ln(x+1)]_0^1 - \int_0^1 (x+1) \times \frac{1}{x+1} dx \\ &= [(x+1) \ln(x+1)]_0^1 - \int_0^1 1 dx \\ &= [(x+1) \ln(x+1)]_0^1 - [x]_0^1 \\ &= (2 \ln(2) - 0) - (1 - 0) \\ &= 2 \ln(2) - 1 \end{aligned}$$

(5) لنحسب $\int_e^{e^2} \frac{\ln x}{\sqrt{x}} dx$:

لدينا : $\int_e^{e^2} \frac{\ln x}{\sqrt{x}} dx = \int_e^{e^2} \frac{1}{\sqrt{x}} \times \ln x dx$

$$\begin{cases} U'(x) = \frac{1}{\sqrt{x}} \\ V(x) = \ln(x) \end{cases} \quad \nearrow \quad \begin{cases} U(x) = 2\sqrt{x} \\ V'(x) = \frac{1}{x} \end{cases} \quad \Downarrow$$

$$\begin{aligned}
 \int_e^{e^2} \frac{\ln x}{\sqrt{x}} dx &= \left[2\sqrt{x} \ln x \right]_e^{e^2} - \int_e^{e^2} 2\sqrt{x} \times \frac{1}{x} dx \\
 &= \left[2\sqrt{x} \ln x \right]_e^{e^2} - 2 \int_e^{e^2} \frac{1}{\sqrt{x}} dx \\
 &= \left[2\sqrt{x} \ln x \right]_e^{e^2} - 2 \left[2\sqrt{x} \right]_e^{e^2} \\
 &= \left(2\sqrt{e^2} \ln(e^2) - 2\sqrt{e} \ln e \right) - 2 \left(2\sqrt{e^2} - 2\sqrt{e} \right) \\
 &= \left(4e - 2\sqrt{e} \right) - 2 \left(2e - 2\sqrt{e} \right) \\
 &= 4e - 2\sqrt{e} - 4e + 4\sqrt{e} \\
 &= 2\sqrt{e}
 \end{aligned}$$

(6) لنحسب $\int_0^1 x^2 e^x dx$:

$$\begin{aligned}
 \begin{cases} U'(x) = e^x \\ V(x) = x^2 \end{cases} &\swarrow \begin{cases} U(x) = e^x \\ V'(x) = 2x \end{cases} \updownarrow \\
 \int_0^1 x^2 e^x dx &= \left[x^2 e^x \right]_0^1 - \int_0^1 2x e^x dx \\
 &= \left[x^2 e^x \right]_0^1 - 2 \int_0^1 x e^x dx \\
 &= e - 2 \int_0^1 x e^x dx
 \end{aligned}$$

لنحسب $\int_0^1 x e^x dx$:

$$\begin{aligned}
 \begin{cases} h'(x) = e^x \\ g(x) = x \end{cases} &\swarrow \begin{cases} h(x) = e^x \\ g'(x) = 1 \end{cases} \updownarrow \\
 \int_0^1 x e^x dx &= \left[x e^x \right]_0^1 - \int_0^1 e^x dx \\
 &= \left[x e^x \right]_0^1 - \left[e^x \right]_0^1 \\
 &= (e - 0) - (e - 1) = 1
 \end{aligned}$$

$$\int_0^1 x^2 e^x dx = e - 2 \quad \text{و بالتالي :}$$

تصحيح التمرين 6

(1)

$$\begin{aligned}
 A &= \int_1^4 |f(x)| dx \cdot (U.A) \\
 &= \int_1^4 \left| \frac{2}{x} \right| dx \cdot (U.A) \\
 &= \int_1^4 \frac{2}{x} dx \cdot (U.A) \\
 &= 2 \int_1^4 \frac{1}{x} dx \cdot (U.A) \\
 &= 2 [\ln x]_1^4 \cdot (U.A) \\
 &= 2 (\ln(4) - \ln(1)) \cdot (U.A) \\
 &= 2 \ln(4) \cdot (U.A)
 \end{aligned}$$

(2)

$$\begin{aligned}
 A &= \int_{\ln 2}^{\ln 4} |f(x)| dx \cdot \|\vec{i}\| \cdot \|\vec{j}\| \\
 &= \int_{\ln 2}^{\ln 4} |1 - e^x| dx \cdot 2cm \cdot 2cm \\
 &= \int_{\ln 2}^{\ln 4} (e^x - 1) dx \cdot 4cm^2 \\
 &= \left[e^x - x \right]_{\ln 2}^{\ln 4} \cdot 4cm^2 \\
 &= ((4 - \ln 4) - (2 - \ln 2)) \cdot 4cm^2 \\
 &= (2 - \ln 2) \cdot 4cm^2 \\
 &= (8 - 4 \ln(2)) cm^2
 \end{aligned}$$

(3)

$$\begin{aligned}
 A &= \int_1^3 |f(x)| dx \cdot \|\vec{i}\| \cdot \|\vec{j}\| \\
 &= \int_1^3 |x^2 - 2x| dx \cdot 1cm \cdot 1cm \\
 &= \left(\int_1^2 |x^2 - 2x| dx + \int_2^3 |x^2 - 2x| dx \right) cm^2 \\
 &= \left(\int_1^2 (2x - x^2) dx + \int_2^3 (x^2 - 2x) dx \right) cm^2 \\
 &= \left(\left[x^2 - \frac{x^3}{3} \right]_1^2 + \left[\frac{x^3}{3} - x^2 \right]_2^3 \right) cm^2 \\
 &= \left(\left(4 - \frac{8}{3} \right) - \left(1 - \frac{1}{3} \right) + (9 - 9) - \left(\frac{8}{3} - 4 \right) \right) cm^2 \\
 &= 2cm^2
 \end{aligned}$$

(4)

$$\begin{aligned}
 A &= \int_0^{\ln 2} |f(x) - g(x)| dx \cdot \|\vec{i}\| \cdot \|\vec{j}\| \\
 &= \int_0^{\ln 2} \left| \frac{2e^x}{e^x + 1} \right| dx \cdot 2cm \cdot 3cm \\
 &= \int_0^{\ln 2} \left(\frac{2e^x}{e^x + 1} \right) dx \cdot 6cm^2 \\
 &= 2 \int_0^{\ln 2} \left(\frac{(e^x + 1)'}{e^x + 1} \right) dx \cdot 6cm^2 \\
 &= 12 \cdot \left[\ln |e^x + 1| \right]_0^{\ln 2} \cdot cm^2 \\
 &= 12 (\ln(3) - \ln(2)) \cdot cm^2 \\
 &= 12 \ln \left(\frac{3}{2} \right) cm^2
 \end{aligned}$$

تصحيح التمرين 7:

$$\begin{aligned}
 V &= \int_0^1 \pi f^2(x) dx \cdot (UV) \\
 &= \pi \int_0^1 x(e^x - 1) dx \\
 &= \pi \int_0^1 (xe^x - x) dx \cdot (UV) \\
 &= \pi \left(\int_0^1 xe^x dx - \int_0^1 x dx \right) \cdot (UV) \\
 &= \pi \left(1 - \left[\frac{x^2}{2} \right]_0^1 \right) \cdot (UV) \\
 &= \pi \left(1 - \left(\frac{1}{2} \right) \right) \cdot (UV) \\
 &= \frac{\pi}{2} (UV)
 \end{aligned}$$

(بالنسبة للتكامل $\int_0^1 xe^x dx$ انظر تصحيح التمرين 5 السؤال 6 باستعمال مكاملة بالأجزاء : $\int_0^1 xe^x dx = 1$)

تصحيح التمرين 8:

(1)

$$\int_1^2 \left(x^4 - \frac{1}{4}x^3 + 2x - 5 - \frac{1}{x} + \frac{4}{\sqrt{x}} \right) dx = \left[\frac{x^5}{5} - \frac{x^4}{16} + x^2 - 5x + 8\sqrt{x} \right]_1^2$$
$$= 8\sqrt{2} - \ln(2) - \frac{379}{80}$$

(2)

$$\int_0^1 3x (x^2 - 1)^4 dx = \frac{3}{2} \int_0^1 2x (x^2 - 1)^4 dx$$
$$= \frac{3}{2} \int_0^1 (x^2 - 1)' (x^2 - 1)^4 dx$$
$$= \frac{3}{2} \left[\frac{(x^2 - 1)^5}{5} \right]_0^1$$
$$= \frac{3}{10}$$

(3)

$$\int_0^1 \frac{x^2}{\sqrt{x^3 + 1}} dx = \frac{1}{3} \int_0^1 \frac{3x^2}{\sqrt{x^3 + 1}} dx$$
$$= \frac{1}{3} \int_0^1 \frac{(x^3 + 1)'}{\sqrt{x^3 + 1}} dx$$
$$= \frac{1}{3} \left[2\sqrt{x^3 + 1} \right]_0^1$$
$$= \frac{1}{3} (2\sqrt{2} - 2)$$
$$= \frac{2}{3} (\sqrt{2} - 1)$$

(4)

$$\begin{aligned}\int_0^1 \frac{x^5}{\sqrt[3]{x^6+1}} dx &= \frac{1}{6} \int_0^1 6x^5 (x^6+1)^{-\frac{1}{3}} dx \\&= \frac{1}{6} \int_0^1 (x^6+1)' (x^6+1)^{-\frac{1}{3}} dx \\&= \frac{1}{6} \left[\frac{(x^6+1)^{\frac{2}{3}}}{\frac{2}{3}} \right]_0^1 \\&= \frac{1}{6} \left[\frac{3}{2} \sqrt[3]{(x^6+1)^2} \right]_0^1 \\&= \frac{1}{6} \left(\frac{3}{2} \sqrt[3]{4} - \frac{3}{2} \right) \\&= \frac{\sqrt[3]{4}-1}{4}\end{aligned}$$

$$\int_1^e \frac{\ln^2(x)}{x} dx = \int_1^e \ln'(x) \ln^2(x) dx = \left[\frac{\ln^3(x)}{3} \right]_1^e = \frac{1}{3} \quad (5)$$

(6)

$$\begin{aligned}\int_2^3 \frac{x^3}{(x^4-1)^2} dx &= \frac{1}{4} \int_2^3 \frac{4x^3}{(x^4-1)^2} dx \\&= \frac{1}{4} \int_2^3 \frac{(x^4-1)'}{(x^4-1)^2} dx \\&= \frac{1}{4} \left[\frac{-1}{x^4-1} \right]_2^3 \\&= \frac{19}{960}\end{aligned}$$

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int_1^4 (\sqrt{x})' e^{\sqrt{x}} dx = 2 \left[e^{\sqrt{x}} \right]_1^4 = 2(e^2 - e) \quad (7)$$

(8)

$$\begin{aligned}\int_0^1 \frac{x+2}{x^2+4x+3} dx &= \frac{1}{2} \int_0^1 \frac{2x+4}{x^2+4x+3} dx \\ &= \frac{1}{2} \int_0^1 \frac{(x^2+4x+3)'}{x^2+4x+3} dx \\ &= \frac{1}{2} \left[\ln |x^2+4x+3| \right]_0^1 \\ &= \frac{1}{2} (\ln 8 - \ln 3) \\ &= \frac{1}{2} \ln \left(\frac{8}{3} \right)\end{aligned}$$

تصحيح التمرين 9:

(1) لنحسب : $\int_0^1 (x+2)e^{-x} dx$

$$\begin{aligned}\begin{cases} U'(x) = e^{-x} \\ V(x) = x+2 \end{cases} &\nearrow \begin{cases} U(x) = -e^{-x} \\ V'(x) = 1 \end{cases} \Downarrow \\ \int_0^1 (x+2)e^{-x} dx &= \left[-(x+2)e^{-x} \right]_0^1 - \int_0^1 -e^{-x} dx \\ &= \left[-(x+2)e^{-x} \right]_0^1 - \left[-e^{-x} \right]_0^1 \\ &= (-3e^{-1}) - (-2) - (e^{-1} - 1) \\ &= 3 - 4e^{-1}\end{aligned}$$

(2) لنحسب : $\int_0^1 x \ln(x+3) dx$

$$\begin{aligned}\begin{cases} U'(x) = x \\ V(x) = \ln(x+3) \end{cases} &\nearrow \begin{cases} U(x) = \frac{x^2}{2} \\ V'(x) = \frac{1}{x+3} \end{cases} \Downarrow\end{aligned}$$

$$\begin{aligned}
 \int_0^1 x \ln(x+3) dx &= \left[\frac{x^2}{2} \ln(x+3) \right]_0^1 - \frac{1}{2} \int_0^1 \frac{x^2}{x+3} dx \\
 &= \left(\frac{1}{2} \ln 4 \right) - (0) - \frac{1}{2} \int_0^1 \frac{x^2 - 3^2 + 3^2}{x+3} dx \\
 &= \frac{1}{2} \ln 4 - \frac{1}{2} \int_0^1 \frac{(x-3)(x+3) + 9}{x+3} dx \\
 &= \frac{1}{2} \ln 4 - \frac{1}{2} \int_0^1 \left(x-3 + \frac{9(x+3)'}{x+3} \right) dx \\
 &= \frac{1}{2} \ln 4 - \frac{1}{2} \left[\frac{x^2}{2} - 3x + 9 \ln|x+3| \right]_0^1 \\
 &= \frac{1}{2} \ln 4 - \left(\left(\frac{-5}{2} + 9 \ln 4 \right) - (9 \ln 3) \right) \\
 &= \frac{5}{4} - 4 \ln 4 + \frac{9}{2} \ln 3
 \end{aligned}$$

(3)

(أ) ليكن x من $\mathbb{R} \setminus \{-1; 0; 1\}$

$$\begin{aligned}
 \frac{-1}{x} + \frac{x}{x^2-1} &= \frac{-(x^2-1) + x^2}{x(x^2-1)} \\
 &= \frac{-x^2 + 1 + x^2}{x(x^2-1)}
 \end{aligned}$$

$$\frac{1}{x(x^2-1)} = \frac{-1}{x} + \frac{x}{x^2-1} : \text{لدينا } \mathbb{R} \setminus \{-1; 0; 1\} \text{ لكل } x$$

(ب)

$$\begin{aligned}
 \int_2^3 \frac{1}{x(x^2-1)} dx &= \int_2^3 \left(\frac{-1}{x} + \frac{x}{x^2-1} \right) dx \\
 &= \int_2^3 \left(\frac{-1}{x} + \frac{1}{2} \times \frac{2x}{x^2-1} \right) dx \\
 &= \int_2^3 \left(\frac{-1}{x} + \frac{1}{2} \times \frac{(x^2-1)'}{x^2-1} \right) dx \\
 &= \left[-\ln x + \frac{1}{2} \ln |x^2-1| \right]_2^3 \\
 &= \left(-\ln 3 + \frac{1}{2} \ln 8 \right) - \left(-\ln 2 + \frac{1}{2} \ln 3 \right) \\
 &= \frac{-3}{2} \ln 3 + \frac{5}{2} \ln 2
 \end{aligned}$$

(ج) لنحسب : $\int_2^3 \frac{x}{(x^2-1)^2} \ln(x) dx$

$$\left\{ \begin{array}{l} U'(x) = \frac{x}{(x^2-1)^2} = \frac{1}{2} \times \frac{2x}{(x^2-1)^2} = \frac{(x^2-1)'}{(x^2-1)^2} \\ V(x) = \ln(x) \end{array} \right. \nearrow \left\{ \begin{array}{l} U(x) = \frac{-1}{2(x^2-1)} \downarrow \\ V'(x) = \frac{1}{x} \end{array} \right.$$

$$\begin{aligned}
 \int_2^3 \frac{x}{(x^2-1)^2} \ln(x) dx &= \left[\frac{-1}{2(x^2-1)} \ln x \right]_2^3 - \int_2^3 \frac{-1}{2x(x^2-1)} dx \\
 &= \left(\frac{-1}{16} \ln 3 \right) - \left(\frac{-1}{6} \ln 2 \right) + \frac{1}{2} \int_2^3 \frac{1}{x(x^2-1)} dx \\
 &= \frac{-1}{16} \ln 3 + \frac{1}{6} \ln 2 - \frac{3}{4} \ln 3 + \frac{5}{4} \ln 2 \\
 &= \frac{17}{12} \ln 2 - \frac{13}{16} \ln 3
 \end{aligned}$$

تصحيح التمرين 10:

(1) لنحسب $\int_0^1 x e^x dx$:

$$\begin{cases} h'(x) = e^x \\ g(x) = x \end{cases} \nearrow \begin{cases} h(x) = e^x \\ g'(x) = 1 \end{cases} \Downarrow$$

$$\begin{aligned} \int_0^1 x e^x dx &= [x e^x]_0^1 - \int_0^1 e^x dx \\ &= [x e^x]_0^1 - [e^x]_0^1 \\ &= (e - 0) - (e - 1) = 1 \end{aligned}$$

(2) لنحسب $\int_0^1 x^2 e^x dx$:

$$\begin{cases} U'(x) = e^x \\ V(x) = x^2 \end{cases} \nearrow \begin{cases} U(x) = e^x \\ V'(x) = 2x \end{cases} \Downarrow$$

$$\begin{aligned} \int_0^1 x^2 e^x dx &= [x^2 e^x]_0^1 - \int_0^1 2x e^x dx \\ &= [x^2 e^x]_0^1 - 2 \int_0^1 x e^x dx \\ &= e - 2 \int_0^1 x e^x dx \end{aligned}$$

و منه : $\int_0^1 x^2 e^x dx = e - 2$

(3)

$$\begin{aligned} A &= \int_0^1 |f(x)| dx \times \|\vec{i}\| \times \|\vec{j}\| \\ &= \int_0^1 (x^2 + 4x + 4) e^x dx \times 1,5cm \times 2cm \quad \left((x^2 + 4x + 4) e^x = (x + 2)^2 e^x \geq 0 \right) \\ &= \left(\int_0^1 x^2 e^x dx + 4 \int_0^1 x e^x dx + 4 \int_0^1 e^x dx \right) \times 3cm^2 \\ &= (e - 2 + 4 \times 1 + 4 [e^x]_0^1) \times 3cm^2 \\ &= (e + 2 + 4(e - 1)) \times 3cm^2 \\ &= (5e - 2) \times 3cm^2 \\ &= (15e - 6) cm^2 \end{aligned}$$

تصحيح التمرين 11:

(1)

✓ الدالة F قابلة للاشتقاق على $]0, +\infty[$

✓ ليكن $x \in]0, +\infty[$:

$$\begin{aligned} F'(x) &= \left(\frac{-\ln x}{x} - \frac{1}{x} \right), \\ &= - \left(\frac{\ln x}{x} + \frac{1}{x} \right), \\ &= - \left(\frac{\ln'(x) \times x - \ln(x) \times (x)'}{x^2} - \frac{1}{x^2} \right) \\ &= - \frac{\frac{1}{x} \times x - \ln x}{x^2} + \frac{1}{x^2} \\ &= - \frac{1 - \ln x}{x^2} + \frac{1}{x^2} \\ &= - \frac{1 - \ln x - 1}{x^2} = \frac{\ln x}{x^2} \end{aligned}$$

إذن لكل x من $]0, +\infty[$:

و بالتالي الدالة F دالة أصلية للدالة f على المجال $]0, +\infty[$

(2)

$$\begin{aligned} \int_1^4 \frac{\ln x}{x^2} dx &= \left[\frac{-\ln x}{x} - \frac{1}{x} \right]_1^4 \\ &= \left(\frac{-\ln 4}{4} - \frac{1}{4} \right) - (0 - 1) \\ &= \frac{3 - \ln 4}{4} \end{aligned}$$

تصحيح التمرين 12:

(1) لنبين أن الدالة $H : x \mapsto x \ln x - x$ دالة أصلية للدالة $h : x \mapsto \ln x$ على $]0, +\infty[$

✓ الدالة H قابلة للاشتقاق على $]0, +\infty[$

✓ ليكن $x \in]0, +\infty[$

$$\begin{aligned} H'(x) &= (x \ln x - x)' \\ &= (x)' \ln x + x \ln'(x) - 1 \\ &= \ln x + x \times \frac{1}{x} - 1 \\ &= \ln(x) + 1 - 1 \\ &= \ln x \end{aligned}$$

إذن لكل x من $]0, +\infty[$: $H'(x) = h(x)$

و منه الدالة $H : x \mapsto x \ln x - x$ دالة أصلية للدالة $h : x \mapsto \ln x$ على $]0, +\infty[$

(2)

$$\begin{aligned} \int_1^2 h(x) dx &= [H(x)]_1^2 \\ &= H(2) - H(1) \\ &= (2 \ln 2 - 2) - (1 \ln 1 - 1) \\ &= 2 \ln 2 - 2 - 0 + 1 \\ &= 2 \ln 2 - 1 \end{aligned}$$

$$A = \int_1^2 |h(x)| dx \times \| \vec{i} \| \times \| \vec{j} \| \quad (3)$$

على المجال $[1, 2]$ لدينا : $h(x) = \ln x \geq 0$

$$A = \int_1^2 h(x) dx \times 2cm \times 2cm \quad \text{إذن :}$$

$$A = (2 \ln 2 - 1) \times 4cm^2 \quad \text{إذن :}$$

$$A = (8 \ln 2 - 4)cm^2 \quad \text{و منه :}$$

تصحيح التمرين 13:

لنحسب : $\int_2^4 |\ln(x) - 1| dx$

x	0	e	$+\infty$
$\ln(x)-1$	-	0	+

$$\begin{aligned}
 \int_2^4 |\ln x - 1| dx &= \int_2^e |\ln x - 1| dx + \int_e^4 |\ln x - 1| dx \\
 &= \int_2^e (1 - \ln x) dx + \int_e^4 (\ln x - 1) dx \\
 &= \left[x - (x \ln x - x) \right]_2^e + \left[(x \ln x - x) - x \right]_e^4 \\
 &= \left[2x - x \ln x \right]_2^e + \left[x \ln x - 2x \right]_e^4 \\
 &= 2e - 12 + 10 \ln 2
 \end{aligned}$$

(انظر التمرين 12 لدينا الدالة $H : x \mapsto x \ln x - x$ دالة أصلية للدالة $h : x \mapsto \ln x$ على $]0, +\infty[$)

تصحيح التمرين 14:

(1)

• لنحسب : $A = \int_1^2 (x^3 - 2x + 3) dx$

$$\begin{aligned}
 A &= \int_1^2 (x^3 - 2x + 3) dx \\
 &= \left[\frac{x^4}{4} - x^2 + 3x \right]_1^2 \\
 &= (4 - 4 + 6) - \left(\frac{1}{4} - 1 + 3 \right) \\
 &= \frac{15}{4}
 \end{aligned}$$

• لنحسب : $B = \int_0^1 x (x^2 + 1)^2 dx$

$$\begin{aligned}
 B &= \int_0^1 x (x^2 + 1)^2 dx \\
 &= \frac{1}{2} \int_0^1 2x (x^2 + 1) dx \\
 &= \frac{1}{2} \int_0^1 (x^2 + 1)' (x^2 + 1) dx \\
 &= \frac{1}{2} \left[\frac{(x^2 + 1)^3}{3} \right]_0^1 \\
 &= \frac{1}{2} \left(\frac{8}{3} - \frac{1}{3} \right) \\
 &= \frac{7}{6}
 \end{aligned}$$

• لنحسب : $C = \int_0^2 \frac{1}{x+1} dx$

$$\begin{aligned} C &= \int_0^2 \frac{1}{x+1} dx \\ &= \int_0^2 \frac{(x+1)'}{x+1} dx \\ &= [\ln|x+1|]_0^2 \\ &= \ln 3 - \ln 1 \\ &= \ln 3 \end{aligned}$$

$$D = \int_{\frac{1}{e}}^e \frac{|\ln x|}{x} dx \quad \text{لنحسب : (2)}$$

x	0	$1 + \infty$
$\ln(x)$	-	+

$$\begin{aligned} \int_{\frac{1}{e}}^e \frac{|\ln x|}{x} dx &= \int_{\frac{1}{e}}^1 \frac{|\ln x|}{x} dx + \int_1^e \frac{|\ln x|}{x} dx \\ &= \int_{\frac{1}{e}}^1 -\frac{1}{x} \ln x dx + \int_1^e \frac{1}{x} \ln x dx \\ &= -\int_{\frac{1}{e}}^1 \ln' x \ln x dx + \int_1^e \ln' x \ln x dx \\ &= -\left[\frac{\ln^2 x}{2} \right]_{\frac{1}{e}}^1 + \left[\frac{\ln^2 x}{2} \right]_1^e \\ &= -\left(0 - \frac{1}{2} \right) + \left(\frac{1}{2} - 0 \right) \\ &= 1 \end{aligned}$$

(3)

(أ) ليكن $x \in \mathbb{R} \setminus \{-1\}$:

$$\begin{aligned} x - 1 + \frac{1}{x+1} &= \frac{(x-1)(x+1)+1}{x+1} \\ &= \frac{x^2 - 1 + 1}{x+1} \\ &= \frac{x^2}{x+1} \end{aligned}$$

$$(\forall x \in \mathbb{R} \setminus \{-1\}) : \frac{x^2}{x+1} = x - 1 + \frac{1}{x+1} \quad \text{إذن :}$$

$$E = \int_0^2 \frac{x^2}{x+1} dx \quad \text{ب) لنحسب}$$

$$\begin{aligned}
 E &= \int_0^2 \frac{x^2}{x+1} dx \\
 &= \int_0^2 \left(x - 1 + \frac{1}{x+1} \right) dx \\
 &= \int_0^2 \left(x - 1 + \frac{(x+1)'}{x+1} \right) dx \\
 &= \left[\frac{x^2}{2} - x + \ln|x+1| \right]_0^2 \\
 &= \ln 3
 \end{aligned}$$

(ج) لنحسب : $F = \int_0^2 x \ln(x+1) dx$

$$\begin{cases} U'(x) = x \\ V(x) = \ln(x+1) \end{cases} \nearrow \begin{cases} U(x) = \frac{x^2}{2} \\ V'(x) = \frac{1}{x+1} \end{cases} \updownarrow$$

$$\begin{aligned}
 F &= \int_0^2 x \ln(x+1) dx \\
 &= \left[\frac{x^2}{2} \ln(x+1) \right]_0^2 - \int_0^2 \frac{x^2}{2(x+1)} dx \\
 &= (2 \ln(3) - 0) - \frac{1}{2} \int_0^2 \frac{x^2}{x+1} dx \\
 &= 2 \ln 3 - \frac{1}{2} E \\
 &= 2 \ln 3 - \frac{1}{2} \ln 3 = \frac{3}{2} \ln 3
 \end{aligned}$$