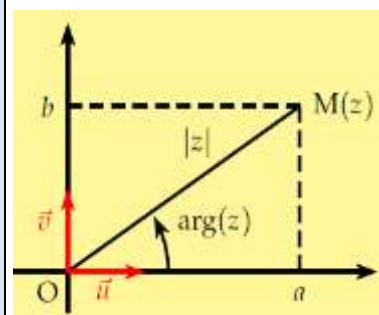


Année scolaire : 2022 – 2023		Résumé des nombres complexes		2 Bac science
Forme algébrique $i^2 = -1$ $Z = a + ib$ avec $(a, b) \in \mathbb{R}^2$ $\text{Re}(z) = a$ et $\text{Im}(z) = b$ $z \in \mathbb{R} \Leftrightarrow \text{Im}(z) = 0$ $z \in i\mathbb{R} \Leftrightarrow \text{Re}(z) = 0$		Conjugué d'un complexe $z = a + ib \Leftrightarrow \bar{z} = a - ib$ $z + \bar{z} = 2\text{Re}(z)$ $z - \bar{z} = 2i\text{Im}(z)$ $\overline{z + z'} = \bar{z} + \bar{z}'$ $\overline{z \times z'} = \bar{z} \times \bar{z}'$; $\overline{z^n} = (\bar{z})^n$ $\overline{\left(\frac{z}{z'}\right)} = \frac{\bar{z}}{\bar{z}'}$ $\bar{\bar{z}} = z \Leftrightarrow z \in i\mathbb{R}$ $\bar{\bar{z}} = z \Leftrightarrow z \in \mathbb{R}$		$[r, \theta] = [r, -\theta] \quad r > 0$ $-[r, \theta] = [r, \theta + \pi]$ $\frac{1}{[r, \theta]} = \left[\frac{1}{r}, -\theta\right]$ $\frac{[r, \theta]}{[r', \theta']} = \left[\frac{r}{r'}, \theta - \theta'\right]$ $[r, \theta]^n = [r^n, n\theta]$ $[r, \theta] \times [r', \theta'] = [r \times r', \theta + \theta']$
Forme trigonométrique : $z = r(\cos \theta + i \sin \theta) \quad r > 0$ $z = [r; \theta] \quad r > 0$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
Forme exponentielle : $z = re^{i\theta} \quad r > 0$ $e^{i\theta} = \cos \theta + i \sin \theta = [1; \theta]$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$(\overrightarrow{AB}, \overrightarrow{AC}) \equiv \arg\left(\frac{Z_C - Z_A}{Z_B - Z_A}\right)[2\pi] \quad Z_A \neq Z_B$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$(\overrightarrow{AB}, \overrightarrow{CD}) \equiv \arg\left(\frac{Z_D - Z_C}{Z_B - Z_A}\right)[2\pi] \quad Z_A \neq Z_B$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$z_I = \frac{Z_A + Z_B}{2} \Leftrightarrow I$ milieu de $[AB]$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$\frac{Z_C - Z_A}{Z_B - Z_A} \in \mathbb{R} \Leftrightarrow A, B$ et C sont alignés ($A \neq B$)		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
Nature d'un triangle		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$\frac{Z_C - Z_A}{Z_B - Z_A} = \left[1; \pm \frac{\pi}{2}\right] \Leftrightarrow ABC$ rectangle et isocèle en A		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$\frac{Z_C - Z_A}{Z_B - Z_A} = \left[1; \pm \frac{\pi}{3}\right] \Leftrightarrow ABC$ triangle équilatéral		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$\frac{Z_C - Z_A}{Z_B - Z_A} = [1; \theta] \Leftrightarrow ABC$ triangle isocèle en A		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$\frac{Z_C - Z_A}{Z_B - Z_A} = \left[r; \pm \frac{\pi}{2}\right] ; r > 0 \Leftrightarrow ABC$ rectangle en A		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
Représentation complexe		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
M, M', Ω et $\vec{u}$ d'affixes respectives z, z', ω et a		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
Translation $t_u : t_u(M) = M' \Leftrightarrow z' = z + a$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
Homothétie $h(\Omega, k) : h(M) = M' \Leftrightarrow z' - \omega = k(z - \omega)$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
Rotation $R(\Omega, \theta) : R(M) = M' \Leftrightarrow z' - \omega = e^{i\theta}(z - \omega)$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
Orthogonalité de deux droites		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$(AB) \perp (CD) \Leftrightarrow \arg\left(\frac{Z_D - Z_C}{Z_B - Z_A}\right) \equiv \pm \frac{\pi}{2} [2\pi]$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
L'ensemble des points		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$ Z - Z_A = Z - Z_B $ L'ensemble des points $M(z)$ est la médiatrice du segment $[AB]$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$
$ Z - Z_A = r$ L'ensemble des points $M(z)$ est le cercle de centre A et de rayon $r \quad r > 0$		Module d'un complexe $ a + ib = \sqrt{a^2 + b^2}$ $ z = -z = \bar{z} $ $ z \times z' = z \times z' $ $ z^n = z ^n$ $\left \frac{z}{z'}\right = \frac{ z }{ z' } \quad z' \neq 0$ $ z + z' \leq z + z' $ $AB = Z_B - Z_A $		Argument d'un complexe $\arg(z) \equiv (\vec{u}; \overrightarrow{OM})[2\pi]$ $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$ $\arg(\bar{z}) \equiv -\arg z [2\pi]$ $\arg(z^n) \equiv n \arg z [2\pi]$ $\arg\left(\frac{z}{z'}\right) \equiv \arg z - \arg z' [2\pi]$ $\arg\left(\frac{1}{z}\right) \equiv -\arg z [2\pi]$ $\arg(-z) \equiv \pi + \arg z [2\pi]$



Moivre

$$\cos \theta + i \sin \theta = [1, \theta]$$

$$(\cos \theta + i \sin \theta)^n = [1, n\theta]$$

$$[1, \theta]^n = [1, n\theta]$$

$$\text{Re}[1, \theta]^n = \cos(n\theta)$$

$$\text{Im}[1, \theta]^n = \sin(n\theta)$$

Equation de second degré dans $\mathbb{C}$

$az^2 + bz + c = 0$ où a, b et c des réels $a \neq 0$	
Solutions de l'équation	$\Delta = b^2 - 4ac$
$z = -\frac{b}{2a}$	$\Delta = 0$
$z_1 = \frac{-b + \sqrt{\Delta}}{2a}$ et $z_2 = \frac{-b - \sqrt{\Delta}}{2a}$	$\Delta > 0$
$z_1 = \frac{-b + i\sqrt{-\Delta}}{2a}$ et $z_2 = \frac{-b - i\sqrt{-\Delta}}{2a}$	$\Delta < 0$

