

السنة 1 بكالوريا علوم تجريبية	الحساب المثلثي حلول مقترحة	سلسلة 2
تمرين 1 :		
1	$\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$	
2	$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\tan\left(\frac{\pi}{12}\right) = \frac{\sin\left(\frac{\pi}{12}\right)}{\cos\left(\frac{\pi}{12}\right)} = \frac{\frac{\sqrt{6} - \sqrt{2}}{4}}{\frac{\sqrt{6} + \sqrt{2}}{4}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \frac{(\sqrt{6} - \sqrt{2})^2}{6 - 2} = \frac{8 - 2\sqrt{12}}{4} = \frac{8 - 4\sqrt{3}}{4} = 2 - \sqrt{3}$	
يمكنك أيضا حساب $\tan\left(\frac{\pi}{12}\right)$ باستعمال الخاصية: $\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$		
تمرين 2 :		
1	لدينا:	$\sqrt{2} \cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right) = \sqrt{2} \left(\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{8}\right) \right)$ $\sqrt{2} \cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right) = \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos\left(\frac{\pi}{8}\right) + \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{8}\right) \right) = \cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)$
2	لدينا:	$\sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{8}\right) \quad \text{منه} \quad \sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right)$ $\sin\left(\frac{\pi}{8}\right) = (\sqrt{2} - 1)\cos\left(\frac{\pi}{8}\right) \quad \text{منه} \quad \sin\left(\frac{\pi}{8}\right) = \sqrt{2} \cos\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right)$ <p>بالتالي: $\frac{\sin\left(\frac{\pi}{8}\right)}{\cos\left(\frac{\pi}{8}\right)} = \sqrt{2} - 1$ أي: $\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$</p>
يمثل التمرين طريقة أخرى لحساب قيمة $\tan\left(\frac{\pi}{8}\right)$		
تمرين 3 :		
$b \in \left] \frac{\pi}{2}; \pi \right[$ ، $a \in \left[0; \frac{\pi}{2} \right[$ ، $\sin b = \frac{3}{7}$ ، $\cos a = \frac{1}{4}$		
نعلم أن: $\sin^2 a + \cos^2 a = 1$ منه: $\sin^2 a + \left(\frac{1}{4}\right)^2 = 1$ منه: $\sin^2 a = 1 - \frac{1}{16} = \frac{15}{16}$		
وبما أن: $a \in \left[0; \frac{\pi}{2} \right[$ فإن: $\sin a > 0$ بالتالي: $\sin a = \frac{\sqrt{15}}{4}$		
نعلم أن: $\sin^2 b + \cos^2 b = 1$ منه: $\left(\frac{3}{7}\right)^2 + \cos^2 b = 1$ منه: $\cos^2 b = 1 - \frac{9}{49} = \frac{40}{49}$		
وبما أن: $b \in \left] \frac{\pi}{2}; \pi \right[$ فإن: $\cos b < 0$ بالتالي: $\cos b = -\frac{\sqrt{40}}{7}$		

$\cos 2a = 2\cos^2 a - 1 = 2 \times \frac{1}{16} - 1 = \frac{1}{8} - 1 = -\frac{7}{8}$ $\sin 2a = 2\sin a \cos a = 2 \times \frac{\sqrt{15}}{4} \times \frac{1}{4} = \frac{\sqrt{15}}{8}$	$\cos 2b = 2\cos^2 b - 1 = 2 \times \frac{40}{49} - 1 = \frac{80}{49} - 1 = \frac{31}{49}$ $\sin 2b = 2\sin b \cos b = 2 \times \frac{3}{7} \times \frac{-\sqrt{40}}{7} = -\frac{6\sqrt{40}}{49}$
تمرين 4 :	
$\begin{aligned}\cos(x+y)\cos(x-y) &= (\cos x \cos y - \sin x \sin y)((\cos x \cos y + \sin x \sin y)) \\ &= (\cos x \cos y)^2 - (\sin x \sin y)^2 = \cos^2 x \cos^2 y - \sin^2 x \sin^2 y \\ &= \cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y \\ &= \cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y \\ &= \cos^2 x - \sin^2 y\end{aligned}$	1
$\begin{aligned}\sin(x+y)\sin(x-y) &= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\ &= (\sin x \cos y)^2 - (\cos x \sin y)^2 \\ &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\ &= (1 - \cos^2 x) \cos^2 y - \cos^2 x (1 - \cos^2 y) \\ &= \cos^2 y - \cos^2 x \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y \\ &= \cos^2 y - \cos^2 x\end{aligned}$	2
تمرين 5 :	
$\begin{aligned}\sin 3x + \sin 5x &= 2\sin\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right) \\ \sin 3x + \sin 5x &= 2\sin(4x)\cos(-x)\end{aligned}$	$\begin{aligned}\cos x + \cos 2x &= 2\cos\left(\frac{x+2x}{2}\right)\cos\left(\frac{x-2x}{2}\right) \\ \cos x + \cos 2x &= 2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{-x}{2}\right)\end{aligned}$
$\begin{aligned}\sin x - \sin \frac{x}{2} &= 2\cos\left(\frac{x+\frac{x}{2}}{2}\right)\sin\left(\frac{x-\frac{x}{2}}{2}\right) \\ \sin x - \sin \frac{x}{2} &= 2\cos\left(\frac{3x}{4}\right)\sin\left(\frac{x}{4}\right)\end{aligned}$	$\begin{aligned}\cos 3x - \cos 7x &= -2\sin\left(\frac{3x+7x}{2}\right)\sin\left(\frac{3x-7x}{2}\right) \\ \cos 3x - \cos 7x &= -2\sin(5x)\sin(-2x)\end{aligned}$
$\begin{aligned}\cos a + \cos b &= 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \\ \sin a + \sin b &= 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)\end{aligned}$	$\begin{aligned}\cos a - \cos b &= -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right) \\ \sin a - \sin b &= 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)\end{aligned}$

🌿 نذكر بقواعد التعميل :