أكاديمية الجهة الشرقية

تمارين محلولة:الحساب المثلثي

المستوى : الأولى باك علوم تجريبية

الأستاذ: نجيب عثماني

1)قواعد مهمة :يجب حفظها

① $\cos(a-b)=\cos a\cos b+\sin a\sin b$

② $\cos(a+b)=\cos a\cos b-\sin a\sin b$

③ $\sin(a+b)=\sin a\cos b+\sin b\cos a$

④ $\sin(a-b)=\sin a\cos b-\sin b\cos a$

(2

⑤ $\tan(a+b)=\dfrac{\tan a+\tan b}{1-\tan a\times\tan b}$

⑥ $\tan(a-b)=\dfrac{\tan a-\tan b}{1+\tan a\times\tan b}$

(3 $\cos(2a)=\cos^2 a-\sin^2 a$ و $\cos(2a)=1-2\sin^2 a$

$\cos(2a)=2\cos^2 a-1$ اذن : $\cos^2 a=\dfrac{1+\cos 2a}{2}$

$\sin^2 a=\dfrac{1-\cos 2a}{2}$ و $\cos^2 a+\sin^2 a=1$

$\sin(2a)=2\sin a\times\cos a$ و $1+\tan^2 a=\dfrac{1}{\cos^2 a}$

$\tan(2a)=\dfrac{2\tan a}{1-\tan^2 a}$ $\quad$ $\tan(x)=\dfrac{2\tan\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}$

(4 $\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}$ و $\sin x=2\sin\frac{x}{2}\times\cos\frac{x}{2}$

$\cos x=\dfrac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$ و $\sin x=\dfrac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

5)بوضع : $t=\tan\left(\frac{x}{2}\right)$

نجد : $\sin x=\dfrac{2t}{1+t^2}$ و $\cos x=\dfrac{1-t^2}{1+t^2}$ و $\tan x=\dfrac{2t}{1-t^2}$

6)قواعد لكيفية تحويل جداء إلى مجموع:

$$\cos a\cos b=\frac{1}{2}\left[\cos(a+b)+\cos(a-b)\right]$$

$$\sin a\sin b=-\frac{1}{2}\left[\cos(a+b)-\cos(a-b)\right]$$

$$\sin a\cos b=\frac{1}{2}\left[\sin(a+b)+\sin(a-b)\right]$$

$$\cos a\sin b=-\frac{1}{2}\left[\sin(a+b)-\sin(a-b)\right]$$

7)قواعد لكيفية تحويل مجموع إلى جداء:

$$\cos p+\cos q=2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$\cos p-\cos q=-2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

$$\sin p+\sin q=2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$\sin p-\sin q=2\cos\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

تمرين1:

أحسب $\cos\frac{\pi}{12}$ و $\sin\frac{\pi}{12}$

أجوبة: $\cos\frac{\pi}{12}=\cos\left(\frac{4\pi-3\pi}{12}\right)=\cos\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)=\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

يمكننا استعمال نتائج الجدول التالي:

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

$$\cos\frac{\pi}{12}=\frac{1}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}=\frac{\sqrt{2}+\sqrt{6}}{4}$$

④ $\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\cos\frac{\pi}{3}$

$$\sin\frac{\pi}{12}=\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\times\frac{1}{2}=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

تمرين2:

أحسب $\tan\frac{\pi}{12}$

الجواب: $\tan\frac{\pi}{12}=\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\times\tan\frac{\pi}{4}}=\dfrac{\sqrt{3}-1}{1+\sqrt{3}}$

$$\tan\frac{\pi}{12}=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-1^2}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$$

تمرين3:

1. أحسب $\cos\frac{5\pi}{12}$ و $\sin\frac{5\pi}{12}$ و $\tan\frac{5\pi}{12}$
2. أحسب $\cos\frac{7\pi}{12}$ و $\sin\frac{7\pi}{12}$ و $\tan\frac{7\pi}{12}$
3. بين أن : $\cos x=\cos\left(x+\frac{\pi}{3}\right)+\cos\left(x-\frac{\pi}{3}\right)$

أجوبة: (1 $\cos\frac{5\pi}{12}=\cos\left(\frac{2\pi+3\pi}{12}\right)=\cos\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)=\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$$\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)=\cos\frac{\pi}{6}\cos\frac{\pi}{4}-\sin\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$\cos\frac{5\pi}{12}=\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}-\frac{1}{2}\times\frac{\sqrt{2}}{2}=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

④ $\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)=\sin\frac{\pi}{6}\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos\frac{\pi}{6}$

$$\sin\frac{5\pi}{12}=\frac{1}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan\frac{5\pi}{12}=\frac{\sin\frac{5\pi}{12}}{\cos\frac{5\pi}{12}}=\frac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}}=\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}=\frac{\left(\sqrt{6}+\sqrt{2}\right)^2}{6-2}=\frac{\left(\sqrt{6}+\sqrt{2}\right)^2}{4}$$

$$\tan\frac{5\pi}{12}=\frac{\left(\sqrt{6}+\sqrt{2}\right)^2}{4}=\frac{8+2\sqrt{12}}{4}=\frac{8+4\sqrt{3}}{4}=2+\sqrt{3}$$

(2 $\cos\frac{7\pi}{12}=\cos\left(\frac{4\pi+3\pi}{12}\right)=\cos\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right)=\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$$\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\cos\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$\cos\frac{7\pi}{12}=\frac{1}{2}\times\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}=\frac{\sqrt{2}-\sqrt{6}}{4}$$

④ $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\sin\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos\frac{\pi}{3}$

$$\sin\frac{7\pi}{12}=\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\times\frac{1}{2}=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan\frac{7\pi}{12}=\frac{\sin\frac{7\pi}{12}}{\cos\frac{7\pi}{12}}=\frac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}}=\frac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}}=\frac{\left(\sqrt{6}+\sqrt{2}\right)^2}{2-6}=\frac{\left(\sqrt{6}+\sqrt{2}\right)^2}{-4}$$

$$\tan\frac{7\pi}{12}=\frac{8+2\sqrt{12}}{-4}=\frac{8+4\sqrt{3}}{-4}=-2-\sqrt{3}$$

(3 $\cos\left(x+\frac{\pi}{3}\right)+\cos\left(x-\frac{\pi}{3}\right)=\cos x$ ؟؟

$$\cos\left(x+\frac{\pi}{3}\right)+\cos\left(x-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}\cos x-\sin\frac{\pi}{3}\sin x+\cos\frac{\pi}{3}\cos x+\sin\frac{\pi}{3}\sin x$$

$$=\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=2\times\frac{1}{2}\cos x=\cos x$$

تمرين4: بين أن : $\sin\left(x+\frac{2\pi}{3}\right)+\sin\left(x-\frac{2\pi}{3}\right)+\sin x=0$

الجواب: لدينا

$$\sin\left(x+\frac{2\pi}{3}\right)=\sin x\cos\frac{2\pi}{3}+\sin\frac{2\pi}{3}\cos x=\sin x\cos\left(\pi-\frac{\pi}{3}\right)+\sin\left(\pi-\frac{\pi}{3}\right)\cos x$$

$$\sin\left(x+\frac{2\pi}{3}\right)=-\sin x\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos x$$

$$\sin\left(x-\frac{2\pi}{3}\right)=\sin x\cos\frac{2\pi}{3}-\sin\frac{2\pi}{3}\cos x=\sin x\cos\left(\pi-\frac{\pi}{3}\right)-\sin\left(\pi-\frac{\pi}{3}\right)\cos x$$

$$\sin\left(x-\frac{2\pi}{3}\right)=-\sin x\cos\frac{\pi}{3}-\sin\frac{\pi}{3}\cos x$$

اذن : $\sin\left(x+\frac{2\pi}{3}\right)+\sin\left(x-\frac{2\pi}{3}\right)+\sin x=-2\sin x\cos\frac{\pi}{3}+\sin x=-\sin x+\sin x=0$

تمرين5: علما أن: $0\prec a\prec\frac{\pi}{2}$ و $0\prec b\prec\frac{\pi}{2}$ و $\cos a=\sin b=\frac{1}{2}$

1. أحسب $\sin a$ و $\cos b$
2. أحسب $\sin(a+b)$

أجوبة:(1 حساب $\cos b$

نعلم أن : $\cos^2 b+\sin^2 b=1$ يعني $\cos^2 b=1-\sin^2 b$ يعني $\cos^2 b=1-\left(\frac{1}{2}\right)^2$

يعني $\cos^2 b=\frac{3}{4}$ يعني $\cos b=\frac{\sqrt{3}}{2}$ أو $\cos b=-\frac{\sqrt{3}}{2}$ ونعلم أن : $0\prec b\prec\frac{\pi}{2}$

اذن : $\cos b=\frac{\sqrt{3}}{2}$

حساب $\sin a$

نعلم أن : $\cos^2 a+\sin^2 a=1$ يعني $\sin^2 a=1-\cos^2 a$ يعني $\sin^2 a=1-\left(\frac{1}{2}\right)^2$

يعني $\sin^2 a=\frac{3}{4}$ يعني $\sin a=\frac{\sqrt{3}}{2}$ أو $\sin a=-\frac{\sqrt{3}}{2}$ ونعلم أن : $0\prec a\prec\frac{\pi}{2}$

اذن : $\sin a=\frac{\sqrt{3}}{2}$

(2 نعلم أن: $\sin(a+b)=\sin a\cos b+\sin b\cos a$

اذن : $\sin(a+b)=\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}+\frac{1}{2}\times\frac{1}{2}=\frac{3}{4}+\frac{1}{4}=1$

تمرين6: علما أن : $\sin x=\frac{1}{3}$ و $x\in\left]0;\frac{\pi}{2}\right[$

أحسب $\cos(2x)$ و $\sin(2x)$

أجوبة: نعلم أن : $\cos(2x)=1-2\sin^2 x$

اذن : $\cos(2x)=1-2\left(\frac{1}{3}\right)^2=1-\frac{2}{9}=\frac{7}{9}$

و نعلم أن : $\sin(2x)=2\sin x\times\cos x$ ومنه يجب حساب $\cos x$:

لدينا : $\cos^2 x+\sin^2 x=1$ يعني $\cos^2 x=1-\sin^2 x$ يعني $\cos^2 b=1-\left(\frac{1}{3}\right)^2$

يعني $\cos^2 x=\frac{8}{9}$ يعني $\cos x=\frac{\sqrt{8}}{3}$ أو $\cos x=-\frac{\sqrt{8}}{3}$ ونعلم أن : $x\in\left]0;\frac{\pi}{2}\right[$

اذن : $\cos x=\frac{\sqrt{8}}{3}$ ومنه : $\sin(2x)=2\frac{1}{3}\times\frac{\sqrt{8}}{3}=\frac{2\sqrt{8}}{9}$

تمرين7: أحسب $\cos\frac{\pi}{8}$ و $\sin\frac{\pi}{8}$ (لاحظ أن $\frac{\pi}{4}=2\times\frac{\pi}{8}$)

أجوبة: حساب : $\cos\frac{\pi}{8}$

نستعمل العلاقة : $\cos^2 a=\frac{1+\cos 2a}{2}$ ونضع مثلا : $a=\frac{\pi}{8}$

ونجد : $\cos^2\frac{\pi}{8}=\frac{1+\cos 2\frac{\pi}{8}}{2}$ يعني $\cos^2\frac{\pi}{8}=\frac{1+\cos\frac{\pi}{4}}{2}$ يعني $\cos^2\frac{\pi}{8}=\frac{2+\sqrt{2}}{4}$

يعني $\cos\frac{\pi}{8}=\sqrt{\frac{2+\sqrt{2}}{4}}$ أو $\cos\frac{\pi}{8}=-\sqrt{\frac{2+\sqrt{2}}{4}}$

ولكننا نعلم أن : $0\leq\frac{\pi}{8}\leq\frac{\pi}{2}$ اذن : $\cos\frac{\pi}{8}\geq 0$ ومنه : $\cos\frac{\pi}{8}=\sqrt{\frac{2+\sqrt{2}}{4}}=\frac{\sqrt{2+\sqrt{2}}}{2}$

حساب: $\sin\frac{\pi}{8}$ يمكننا استعمال احدى القواعد التالية : $\sin^2 a=\frac{1-\cos 2a}{2}$ أو

$\sin(2a)=2\sin a\times\cos a$ أو $\cos^2 a+\sin^2 a=1$

لدينا : $\sin^2 a=\frac{1-\cos 2a}{2}$ ونضع مثلا : $a=\frac{\pi}{8}$

ونجد : $\sin^2\frac{\pi}{8}=\frac{1-\cos 2\frac{\pi}{8}}{2}$ يعني $\sin^2\frac{\pi}{8}=\frac{1-\cos\frac{\pi}{4}}{2}$ يعني $\sin^2\frac{\pi}{8}=\frac{2-\sqrt{2}}{4}$

يعني $\sin\frac{\pi}{8}=\sqrt{\frac{2-\sqrt{2}}{4}}$ أو $\sin\frac{\pi}{8}=-\sqrt{\frac{2-\sqrt{2}}{4}}$

ولكننا نعلم أن : $0\leq\frac{\pi}{8}\leq\frac{\pi}{2}$ اذن : $\sin\frac{\pi}{8}\geq 0$ ومنه : $\sin\frac{\pi}{8}=\sqrt{\frac{2-\sqrt{2}}{4}}=\frac{\sqrt{2-\sqrt{2}}}{2}$

تمرين8: بين أن : $\frac{\sin 3x}{\sin x}-\frac{\cos 3x}{\cos x}=2$ $\forall x\in\left]0;\frac{\pi}{2}\right[$

الجواب : $\frac{\sin 3x}{\sin x}-\frac{\cos 3x}{\cos x}=\frac{\sin 3x\cos x-\sin x\cos 3x}{\sin x\cos x}=\frac{\sin(3x-x)}{\sin x\cos x}$

$$=\frac{\sin(3x-x)}{\sin x\cos x}=\frac{\sin 2x}{\sin x\cos x}=\frac{2\sin x\cos x}{\sin x\cos x}=2$$

تمرين9: علما أن $\tan\left(\frac{x}{2}\right)=3$: أحسب $\tan x$ و $\sin x$ و $\cos x$

الجواب : نستعمل القواعد : $\sin x=\frac{2t}{1+t^2}$ و $\cos x=\frac{1-t^2}{1+t^2}$ و $\tan x=\frac{2t}{1-t^2}$

$\tan x=\frac{2\tan\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}=\frac{2\times3}{1-3^2}=\frac{6}{-8}=-\frac{3}{4}$ و $\sin x=\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}=\frac{2\times3}{1+3^2}=\frac{6}{10}=\frac{3}{5}$

$$\sin x=\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}=\frac{1-3^2}{1+3^2}=\frac{-8}{10}=-\frac{4}{5}$$

تمرين10: بين أن : $\forall x\in\mathbb{R}$

1) $\sin^2 2x-\cos 2x-1=-2\cos^2 x\times\cos 2x$

2) $2\sin^2 x+12\cos^2 x=5\cos 2x+7$

أجوبة: 1) $\sin^2 2x-\cos 2x-1=(2\cos x\sin x)^2-2\cos^2 x+1-1$

$$4\cos^2 x\sin^2 x-2\cos^2 x=-2\cos^2 x\cos 2x$$

2) $2\sin^2 x+12\cos^2 x=2\sin^2 x+12\left(1-\sin^2 x\right)=-10\sin^2 x+12$

$$=\frac{-10}{2}(1-\cos 2x)+12=-5(1-\cos 2x)+12=5\cos 2x+7$$

تمرين11: بين أن : $\forall x\in\mathbb{R}$

1) $\sin 3x=\sin x\times\left(3-4\sin^2 x\right)$

2) $\cos 3x=\cos x\left(4\cos^2 x-3\right)$

3) $\cos(4x)=8\cos^4 x-8\cos^2 x+1$

4) $\sin(4x)=4\sin x\left(2\cos^3 x-\cos x\right)$

5) $\cos^3 x=\frac{1}{4}(3\cos x+\cos 3x)$

أجوبة: 1) $\sin 3x=\sin(2x+x)=\sin 2x\cos x+\cos 2x\sin x$

$$=2\sin x\cos^2 x+\left(1-2\sin^2 x\right)\sin x=2\sin x\left(1-\sin^2 x\right)+\left(1-2\sin^2 x\right)\sin x$$

$$=2\sin x-2\sin^3 x+\sin x-2\sin^3 x=3\sin x-4\sin^3 x=\sin x\left(3-4\sin^2 x\right)$$

2) $\cos 3x=\cos(2x+x)=\cos x\cos 2x-\sin 2x\sin x$

$$=\cos x\left(2\cos^2 x-1\right)+\sin x\times 2\cos x\sin x=2\cos^3 x-\cos x-2\cos x\sin^2 x$$

$$=2\cos^3 x-\cos x-2\cos x\left(1-\cos^2 x\right)=2\cos^3 x-\cos x-2\cos x+2\cos^2 x$$

$$=4\cos^3 x-3\cos x=\cos x\left(4\cos^2 x-3\right)$$

3) $\cos(4x)=\cos(2\times 2x)=2\cos^2 2x-1=2\left(2\cos^2 x-1\right)^2-1$

$$=2\left(4\cos^4 x-4\cos^2 x+1\right)-1=8\cos^4 x-8\cos^2 x+1$$

4) $\sin(4x)=\sin(2\times 2x)=2\sin 2x\cos 2x=2\times 2\sin x\cos x\left(2\cos^2 x-1\right)$

$$\sin(4x)=4\sin x\cos x\left(2\cos^2 x-1\right)=4\sin x\left(2\cos^3 x-\cos x\right)$$

5) $\cos^3 x=\frac{1}{4}(3\cos x+\cos 3x)$ ؟؟

طريقة1:

$$\frac{1}{4}(3\cos x+\cos 3x)=\frac{1}{4}(3\cos x+\cos(x+2x))=\frac{1}{4}(3\cos x+\cos x\cos 2x-\sin x\sin 2x)$$

$$=\frac{1}{4}\left(3\cos x+\cos x\left(2\cos^2 x-1\right)-2\sin x\sin x\cos x\right)$$

$$=\frac{1}{4}\left(2\cos^3 x-\cos x-2\cos x+2\cos^3 x+3\cos x\right)=\frac{1}{4}\left(4\cos^3 x\right)=\cos^3 x$$

طريقة2: نستعمل صيغة تحويل جداء الى مجموع

$$\cos^3 x=\cos^2 x\times\cos x=\frac{1+\cos 2x}{2}\times\cos x=\frac{1}{2}(\cos x+\cos 2x\times\cos x)$$

$$\cos^3 x=\frac{1}{2}\left(\cos x+\frac{1}{2}(\cos 3x+\cos x)\right)=\frac{1}{2}\cos x+\frac{1}{4}\cos 3x+\frac{1}{4}\cos x=\frac{3}{4}\cos x+\frac{1}{4}\cos 3x$$

ومنه : $\cos^3 x=\frac{1}{4}(3\cos x+\cos 3x)$

تمرين12: علما أن : $P(x)=\sin 2x-\sin x$ و $Q(x)=1+\cos x+\cos 2x$

بين أن : $Q(x)=\cos x(2\cos x+1)$ و أن $P(x)=\sin x(2\cos x-1)$

أجوبة :

$$Q(x)=1+\cos x+\cos 2x=1+\cos x+2\cos^2 x-1=\cos x+2\cos^2 x=\cos x(1+2\cos x)$$

$$P(x)=\sin 2x-\sin x=2\sin x\cos x-\sin x=\sin x(2\cos x-1)$$

تمرين13:

أكتب على شكل مجموع :

1) $\cos 2x\times\sin 4x$ 2) $\sin x\times\sin 3x$ 3) $\cos 4x\times\cos 6x$

أجوبة : 1)

$$\cos 2x\times\sin 4x=\frac{1}{2}(\sin(2x+4x)-\sin(2x-4x))=\frac{1}{2}(\sin 6x-\sin(-2x))$$

$$=\frac{1}{2}(\sin 6x+\sin 2x)=\frac{1}{2}\sin 6x+\frac{1}{2}\sin 2x$$

2) $\sin x\times\sin 3x=\frac{1}{2}(\cos(x+3x)-\cos(x-3x))=\frac{1}{2}(\cos 4x-\cos(-2x))$

$$\sin x\times\sin 3x=\frac{1}{2}(\cos 4x-\cos(2x))=\frac{1}{2}\cos 4x-\frac{1}{2}\cos(2x)$$

3) $\cos 4x\times\cos 6x=\frac{1}{2}(\cos(4x+6x)+\cos(4x-6x))=\frac{1}{2}(\cos 4x-\cos(-2x))$

$$\cos 4x\times\cos 6x=\frac{1}{2}\cos 4x-\frac{1}{2}\cos(2x)$$

تمرين14: أكتب على شكل جداء: $\sin 2x+\sin 4x$

الجواب: $\sin 2x+\sin 4x=2\sin\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)$

$$\sin 2x+\sin 4x=2\sin 3x\cos(-2x)=2\sin 3x\cos 2x$$

تمرين15:

1. بين $\sin\frac{3\pi}{11}+\sin\frac{7\pi}{11}=2\sin\left(\frac{5\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)$

2. بين $\sin\frac{3\pi}{11}-\sin\frac{7\pi}{11}=-2\cos\left(\frac{5\pi}{11}\right)\sin\left(\frac{2\pi}{11}\right)$

3. استنتج أن : $\frac{\sin\frac{3\pi}{11}+\sin\frac{7\pi}{11}}{\sin\frac{3\pi}{11}-\sin\frac{7\pi}{11}}=-\frac{\tan\left(\frac{5\pi}{11}\right)}{\tan\left(\frac{2\pi}{11}\right)}$

أجوبة: 1) $\sin\frac{3\pi}{11}+\sin\frac{7\pi}{11}=2\sin\left(\frac{\frac{3\pi}{11}+\frac{7\pi}{11}}{2}\right)\cos\left(\frac{\frac{3\pi}{11}-\frac{7\pi}{11}}{2}\right)$

$$\sin\frac{3\pi}{11}+\sin\frac{7\pi}{11}=2\sin\left(\frac{5\pi}{11}\right)\cos\left(-\frac{2\pi}{11}\right)=2\sin\frac{5\pi}{11}\cos\frac{2\pi}{11}$$

2) $\sin\frac{3\pi}{11}-\sin\frac{7\pi}{11}=2\cos\left(\frac{\frac{3\pi}{11}+\frac{7\pi}{11}}{2}\right)\sin\left(\frac{\frac{3\pi}{11}-\frac{7\pi}{11}}{2}\right)$

$$\sin\frac{3\pi}{11}-\sin\frac{7\pi}{11}=2\cos\left(\frac{5\pi}{11}\right)\sin\left(-\frac{2\pi}{11}\right)=-2\cos\frac{5\pi}{11}\sin\frac{2\pi}{11}$$

(3 $\frac{\sin\frac{3\pi}{11}+\sin\frac{7\pi}{11}}{\sin\frac{3\pi}{11}-\sin\frac{7\pi}{11}}=-\frac{2\sin\left(\frac{5\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)}{-2\cos\left(\frac{5\pi}{11}\right)\sin\left(\frac{2\pi}{11}\right)}$

$$=\frac{\sin\left(\frac{5\pi}{11}\right)}{\cos\left(\frac{5\pi}{11}\right)}\times\frac{\cos\left(\frac{2\pi}{11}\right)}{\sin\left(\frac{2\pi}{11}\right)}=-\tan\left(\frac{5\pi}{11}\right)\times\frac{1}{\tan\left(\frac{2\pi}{11}\right)}=\frac{\tan\left(\frac{5\pi}{11}\right)}{\tan\left(\frac{2\pi}{11}\right)}$$

تمرين16: بين أن : $\frac{\cos 2x-\cos 4x}{\cos 2x+\cos 4x}=\tan 3x\times\tan x$

الجواب: $\cos 2x-\cos 4x=-2\sin\left(\frac{2x+4x}{2}\right)\sin\left(\frac{2x-4x}{2}\right)=2\sin(3x)\sin x$

$$\cos 2x+\cos 4x=-2\cos\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)=2\cos 3x\cos x$$

ملاحظة: $\cos(-x)=\cos x$ و $\sin(-x)=-\sin x$

$$\frac{\cos 2x-\cos 4x}{\cos 2x+\cos 4x}=\frac{2\sin 3x\sin x}{2\cos 3x\cos x}=\frac{\sin 3x}{\cos 3x}\times\frac{\sin x}{\cos x}=\tan 3x\times\tan x$$

تمرين17: بين أن : $\cos^2\frac{5x}{2}-\cos^2\frac{3x}{2}=-\sin 4x\times\sin x$

الجواب: $\cos^2\frac{5x}{2}-\cos^2\frac{3x}{2}=\left(\cos\frac{5x}{2}+\cos\frac{3x}{2}\right)\left(\cos\frac{5x}{2}-\cos\frac{3x}{2}\right)$

$$\cos\frac{5x}{2}+\cos\frac{3x}{2}=2\cos\left(\frac{\frac{5x}{2}+\frac{3x}{2}}{2}\right)\cos\left(\frac{\frac{5x}{2}-\frac{3x}{2}}{2}\right)=2\cos(2x)\cos\left(\frac{x}{2}\right)$$

$$\cos\frac{5x}{2}-\cos\frac{3x}{2}=-2\sin\left(\frac{\frac{5x}{2}+\frac{3x}{2}}{2}\right)\sin\left(\frac{\frac{5x}{2}-\frac{3x}{2}}{2}\right)=-2\sin(2x)\sin\left(\frac{x}{2}\right)$$

ومنه : $\cos^2\frac{5x}{2}-\cos^2\frac{3x}{2}=2\cos(2x)\cos\left(\frac{x}{2}\right)\times-2\sin(2x)\sin\left(\frac{x}{2}\right)$

$$=-2\cos(2x)\times\sin(2x)2\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)=-\sin(4x)\sin x$$

تمرين18: بين أن : $\sin x+\sin 2x+\sin 3x=2\sin x\cos x(1+2\cos x)$

الجواب :

$$\sin x+\sin 2x+\sin 3x=\sin 2x+\sin x+\sin 3x=\sin 2x+2\sin 2x\cos x$$
$$=\sin 2x+2\sin 2x\cos x=\sin 2x(1+2\cos x)=2\sin x\cos x(1+2\cos x)$$

تمرين19: بين أن $\cos x-\sin x=\sqrt{2}\cos\left(\frac{\pi}{4}+x\right)$

الجواب: $a=1$ و $a=-1$.

نحسب : $\sqrt{a^2+b^2}=\sqrt{1^2+(-1)^2}=\sqrt{2}$

$$\cos x-\sin x=\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)=\sqrt{2}\left(\cos\frac{\pi}{4}\cos x-\sin\frac{\pi}{4}\sin x\right)$$

$$\cos x-\sin x=\sqrt{2}\cos\left(\frac{\pi}{4}+x\right)$$

تمرين20: حل في $[0;2\pi]$ المعادلة : $\sqrt{3}\cos x+\sin x=\sqrt{3}$

الجواب: نحول أولا : $\sqrt{3}\cos x+\sin x$

$a=\sqrt{3}$ و $a=1$.

نحسب : $\sqrt{a^2+b^2}=\sqrt{\sqrt{3}^2+1^2}=\sqrt{4}=2$

$$\sqrt{3}\cos x+\sin x=2\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)=2\left(\cos\frac{\pi}{6}\cos x+\sin\frac{\pi}{6}\sin x\right)$$

$$\sqrt{3}\cos x+\sin x=2\cos\left(x-\frac{\pi}{6}\right)$$

$$2\cos\left(x-\frac{\pi}{6}\right)=\sqrt{3}\Leftrightarrow\sqrt{3}\cos x+\sin x=\sqrt{3}$$

$$\cos\left(x-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{6}\right)\Leftrightarrow 2\cos\left(x-\frac{\pi}{6}\right)=\sqrt{3}$$

يعني: $x-\frac{\pi}{6}=\frac{\pi}{6}+2k\pi$ أو $x-\frac{\pi}{6}=-\frac{\pi}{6}+2k\pi$

يعني: $x=\frac{\pi}{3}+2k\pi$ أو $x=2k\pi$

ومنه : $S=\left\{0;\frac{\pi}{3};2\pi\right\}$

الأستاذ : عثماني نجيب