

جناي بنشوايف

التقريب الأول:

$$\lim_{x \rightarrow 3} \frac{2x^2 - 3x - 9}{x^2 + x - 12} = \lim_{x \rightarrow 3} \frac{(x-3)(2x+3)}{(x-3)(x+4)}$$

(1)

$$= \lim_{x \rightarrow 3} \frac{2x+3}{x+4}$$

$$= \boxed{\frac{9}{7}}$$

$$\lim_{x \rightarrow -3} \frac{\sqrt{3-2x^3} - 3}{\sqrt{-3x^3} - 3} = \lim_{x \rightarrow -3} \frac{(\sqrt{3-2x^3} - 3)(\sqrt{3-2x^3} + 3)(\sqrt{-3x^3} + 3)}{(\sqrt{-3x^3} - 3)(\sqrt{-3x^3} + 3)(\sqrt{3-2x^3} + 3)}$$

(2)

$$= \lim_{x \rightarrow -3} \frac{(3-2x-9)(\sqrt{-3x^3} + 3)}{(-3x-9)(\sqrt{3-2x^3} + 3)}$$

$$= \lim_{x \rightarrow -3} \frac{(-2x-6)(\sqrt{-3x^3} + 3)}{(-3x-9)(\sqrt{3-2x^3} + 3)}$$

$$= \lim_{x \rightarrow -3} \frac{-2(x+3)(\sqrt{-3x^3} + 3)}{-3(x+3)(\sqrt{3-2x^3} + 3)}$$

$$= \lim_{x \rightarrow -3} \frac{2}{3} \times \frac{\sqrt{-3x^3} + 3}{\sqrt{3-2x^3} + 3}$$

$$= \frac{2}{3} \times \frac{6}{6} = \boxed{\frac{2}{3}}$$

$$\lim_{\substack{x \rightarrow -5 \\ x > -5}} \frac{x^2 - 5}{x^2 + 5x}$$

(3)

$$\lim_{x \rightarrow -5} x^2 - 5 = 20 \quad \text{لدينا}$$

$$\lim_{x \rightarrow -5} x^2 + 5x = 0 \quad \text{و}$$

جدول إشارة المقام:

x	$-\infty$	-5	0	$+\infty$	
$x^2 + 5x$	+	o	-	o	+

$$\lim_{\substack{x \rightarrow -5 \\ x > -5}} \frac{x^2 - 5}{x^2 + 5x} = -\infty$$

وهنا سيق فان :

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{4x^2 + 2x} - 2x}{x} = \lim_{x \rightarrow +\infty} \frac{4x^2 + 2x - 4x^2}{\sqrt{4x^2 + 2x} + 2x} \quad (1)$$

$$= \lim_{x \rightarrow +\infty} \frac{2x}{x\sqrt{4 + \frac{2}{x}} + 2x}$$

$$= \lim_{x \rightarrow +\infty} \frac{2}{x(\sqrt{4 + \frac{2}{x}} + 2)}$$

$$= \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{4 + \frac{2}{x}} + 2}$$

لدينا $\lim_{x \rightarrow +\infty} \frac{2}{x} = 0$ يعني أن $\lim_{x \rightarrow +\infty} \sqrt{4 + \frac{2}{x}} = 2$ إذن $\lim_{x \rightarrow +\infty} \sqrt{4 + \frac{2}{x}} + 2 = 4$

ومن هنا فإن $\lim_{x \rightarrow +\infty} \frac{2}{\sqrt{4 + \frac{2}{x}} + 2} = \frac{2}{4} = \boxed{\frac{1}{2}}$

المسألة الثانية:

$a = 2$ و $f(x) = x^3 - x^2$ (1)

$$f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \rightarrow 2} \frac{x^3 - x^2 - 4}{x - 2}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + x + 2)}{(x-2)}$$

$$f'(2) = \lim_{x \rightarrow 2} x^2 + x + 2 = \boxed{8}$$

إذن f تقبل الاستيفاق في $a = 2$ و $f'(2) = 8$

$a = -1$ و $f(x) = \sqrt{3x+4} - 3$ (2)

$$f'(-1) = \lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x + 1}$$

$$= \lim_{x \rightarrow -1} \frac{\sqrt{3x+4} - 3 + 2}{x + 1}$$

$$= \lim_{x \rightarrow -1} \frac{\sqrt{3x+4} - 1}{x + 1}$$

$$= \lim_{x \rightarrow -1} \frac{(\sqrt{3x+4} - 1)(\sqrt{3x+4} + 1)}{(x+1)(\sqrt{3x+4} + 1)}$$

$$f'(-1) = \lim_{x \rightarrow -1} \frac{3x+2}{(x+1)(\sqrt{3x+4}+1)}$$

$$= \lim_{x \rightarrow -1} \frac{3(x+1)}{(x+1)(\sqrt{3x+4}+1)}$$

$$f'(-1) = \boxed{\frac{3}{2}}$$

إذن f تقبل الاشتقاق في $a = -1$ و $f'(-1) = \frac{3}{2}$

$$a = 1 \quad \text{و} \quad f(x) = \frac{x^2}{3x-2} \quad (3)$$

$$f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{\frac{x^2}{3x-2} - 1}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{\frac{x^2 - 3x + 2}{3x-2}}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)(x-2)}{(3x-2)(x-1)}$$

$$f'(1) = \lim_{x \rightarrow 1} \frac{x-2}{3x-2} = \boxed{-1}$$

إذن f تقبل الاشتقاق في $a = 1$ و $f'(1) = -1$

التمرين 2: المشتقات

$$f(x) = x^2 - \frac{4}{3}x\sqrt{x^2+1} + 2 \quad (1)$$

$$f'(x) = (x^2)' - \frac{4}{3}(x\sqrt{x^2+1})'$$

$$= 2x - \frac{4}{3}[(x)'\sqrt{x^2+1} + x(\sqrt{x^2+1})']$$

$$= 2x - \frac{4}{3}[\sqrt{x^2+1} + \frac{x}{2\sqrt{x^2+1}}]$$

$$= 2x - \frac{4}{3} \cdot \frac{2\sqrt{x^2+1}}{2}$$

$$= 2x - 2\sqrt{x^2+1}$$

$$\boxed{f'(x) = 2x(\sqrt{x^2+1} - 1)}$$

$$f(x) = (x+2)\sqrt{x-3} + \sqrt{3} \quad (2)$$

$$f'(x) = (x+2)'(\sqrt{x-3}) + (\sqrt{x-3})'(x+2)$$

$$= \sqrt{x-3} + \frac{x+2}{2\sqrt{x-3}}$$

$$= \frac{2(x-3) + (x+2)}{2\sqrt{x-3}}$$

$$f'(x) = \frac{2x-6+x+2}{2\sqrt{x-3}}$$

$$f'(x) = \frac{3x-4}{2\sqrt{x-3}}$$

$$f(x) = \frac{x+2}{\sqrt{x-1}+1} \quad (4)$$

$$f'(x) = \frac{(x+2)'(\sqrt{x-1}+1) - (x+2)(\sqrt{x-1}+1)'}{(\sqrt{x-1}+1)^2}$$

$$f'(x) = \frac{(\sqrt{x-1}+1) - (x+2)\left(\frac{1}{2\sqrt{x-1}}\right)}{(\sqrt{x-1}+1)^2}$$

$$= \frac{\frac{2x-2+2\sqrt{x-1}-(x+2)}{2\sqrt{x-1}}}{(\sqrt{x-1}+1)^2}$$

$$f'(x) = \frac{x+2\sqrt{x-1}-4}{2(\sqrt{x-1}+1)^2\sqrt{x-1}}$$

$$(\sqrt{x-1}-1)(\sqrt{x-1}+3) = x-1+3\sqrt{x-1}-\sqrt{x-1}-3$$

$$(\sqrt{x-1}-1)(\sqrt{x-1}+3) = x+2\sqrt{x-1}-4$$

$$f'(x) = \frac{(\sqrt{x-1}-1)(\sqrt{x-1}+3)}{2(\sqrt{x-1}+1)^2\sqrt{x-1}} \quad \text{وحيث ان}$$

$$f(x) = \frac{2x^3-3x^2}{(x+1)^2} \quad (3)$$

$$f'(x) = \frac{(2x^3-3x^2)'(x+1)^2 - (x+1)^2'(2x^3-3x^2)}{(x+1)^4}$$

$$= \frac{(6x^2-6x)(x+1)^2 - 2(x+1)(2x^3-3x^2)}{(x+1)^4}$$

$$= \frac{6x^2(x+1)^2 - 2(x+1)(2x^3-3x^2)}{(x+1)^4}$$

$$f'(x) = \frac{(x+1)(6x(x-1)(x+1) - 4x^3 + 6x^2)}{(x+1)^4}$$

$$f'(x) = \frac{6x(x^2-1) - 4x^3 + 6x^2}{(x+1)^3}$$

$$= \frac{6x^3 - 6x - 4x^3 + 6x^2}{(x+1)^3}$$

$$= \frac{2x^3 + 6x^2 - 6x}{(x+1)^3}$$

$$f'(x) = \frac{2x(x^2 + 3x - 3)}{(x+1)^3}$$

التمرين الرابع :

1- أ- لنثبت أن $u_n > 0$:

لدينا $u_0 = 2 > 0$ ، $n=0$

- بفرض أن $u_n > 0$ ونثبت أن $u_{n+1} > 0$

لدينا

$$3 + 2u_n > 3 > 0 \text{ ونفرض } u_n > 0$$

$$u_{n+1} = \frac{u_n}{3 + 2u_n} > 0 \text{ لدينا}$$

$$\text{إذن } u_{n+1} = \frac{u_n}{3 + 2u_n} > 0 \text{ وبالتالي فإن } u_n > 0$$

ب- لنثبت أن (u_n) متناقصة :

لدينا

$$u_{n+1} - u_n = \frac{u_n}{3 + 2u_n} - u_n$$

$$= \frac{u_n - 3u_n - 2u_n^2}{3 + 2u_n}$$

$$= \frac{-2u_n^2 - 4u_n}{3 + 2u_n}$$

$$u_{n+1} - u_n = \frac{-2u_n(u_n + 2)}{3 + 2u_n}$$

لدينا $3 + 2u_n > 3 > 0$ و $-2u_n < 0$ و $u_n + 2 > 2 > 0$ أي أن $-2u_n(u_n + 2) < 0$

$$u_{n+1} - u_n = \frac{-2u_n(u_n + 2)}{3 + 2u_n} < 0 \text{ ونستنتج أن}$$

أن (u_n) متناخبة متناقصة .

2 -

أ - لنثبت أن v_n متتالية هندسية أساسها $q = \frac{1}{3}$ وحدها الأول $v_0 = \frac{2}{3}$

لدينا
$$v_n = \frac{u_n}{u_{n+1}} \Leftrightarrow v_{n+1} = \frac{u_{n+1}}{u_{n+2}}$$

يعني
$$v_n = \frac{u_n}{3 + 2u_n} = \frac{\frac{u_n}{3 + 2u_n} + 1}{3 + 2\left(\frac{u_n}{3 + 2u_n} + 1\right)}$$

يعني
$$v_n = \frac{3 + 2u_n}{u_n + 3 + 2u_n} = \frac{3 + 2u_n}{3 + 2u_n} = 1$$

يعني
$$v_n = \frac{1}{3} \times v_{n+1} \Leftrightarrow v_n = \frac{u_n}{3(u_{n+1} + 1)}$$

إذن v_n متتالية هندسية أساسها $\frac{1}{3}$ ولدينا
$$v_0 = \frac{2}{3} \Leftrightarrow v_0 = \frac{u_0}{u_0 + 1} \Leftrightarrow \frac{2}{3} = \frac{u_0}{u_0 + 1}$$

$$v_0 = \frac{2}{3}$$

ب - لتعبر عن v_n بدلالة n ونثبت أن $u_n = \frac{2}{3^{n+1} - 2}$

لدينا
$$v_n = v_0 \cdot \left(\frac{1}{3}\right)^n$$

يعني
$$v_n = \frac{2}{3} \cdot \left(\frac{1}{3}\right)^n$$
 إذن

$$v_n = \frac{2}{3^{n+1}}$$

$$v_n u_n + v_n - u_n = 0$$

$$u_n (v_n - 1) + v_n = 0$$

$$u_n = \frac{v_n}{1 - v_n}$$

وعندئذ
$$u_n = \frac{\frac{2}{3^{n+1}}}{1 - \frac{2}{3^{n+1}}}$$

$$u_n = \frac{2}{3^{n+1} - 2}$$
 إذن

ولدينا
$$v_n = \frac{u_n}{u_{n+1}}$$
 إذن

يعني

يعني

3 - لنثبت أن $S = n + \frac{1}{3^{n+1}}$

لدينا
$$S = \frac{1}{u_0 + 1} + \frac{1}{u_1 + 1} + \dots + \frac{1}{u_n + 1}$$

$$u_n = \frac{v_n}{1-v_n} \quad \text{ولدينا}$$

$$u_{n+1} = \frac{v_{n+1}-v_n}{1-v_n} \quad \text{يعني}$$

$$\left| \frac{1}{u_{n+1}} = 1-v_n \right| \Leftrightarrow \frac{1}{u_{n+1}} = \frac{1-v_n}{1} \quad \text{يعني ان}$$

$$S_n = 1-v_0 + 1-v_1 + \dots + 1-v_n \quad \text{مجموعه}$$

$$S_n = (1+1+\dots+1) - (v_0+v_1+\dots+v_n)$$

$$S_n = (n+1) - v_0 \times \frac{1-q^{n+1}}{1-q}$$

$$S_n = n+1 - \frac{2}{3} \times \frac{1-\frac{1}{3^{n+1}}}{\frac{2}{3}}$$

$$S_n = n+1 - 1 + \frac{1}{3^{n+1}}$$

$$S_n = n + \frac{1}{3^{n+1}}$$